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Three essays on first-price auctions with  
independent private values

By

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THESIS

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## Statement of Originality

I certify that this work contains no material which has been accepted for the award of any other degree or diploma in my name, in any university or other tertiary institution and to the best of my knowledge and belief, contains no material previously published or written by another person, except where due reference has been made in the text. In addition, I certify that no part of this work will, in the future, be used in a submission in my name, for any other degree or diploma in any university or other tertiary institution without the prior approval of the University of Adelaide and, where applicable, any partner institution responsible for the joint-award of this degree.

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## List of Abbreviations

Adlab = Adelaide Laboratory for Experimental Economics

BDM = Becker-DeGroot-Marschak

CE = Certainty Equivalent

CRRA = Constant Relative Risk Aversion

CRRAM = Constant Relative Risk Aversion Model

IPV = Independent Private Value

KIFA = Kunming International Flower Auction

LASNE = Loss Averse Symmetric Nash Equilibrium

NDARA = Non-Decreasing Absolute Risk Aversion

OLS = Ordinary Least Squares

ORSEE = Online Recruitment System for Economic Experiments

QRE = Quantal Response Equilibrium

RASNE = Risk Averse Symmetric Nash Equilibrium

RNNE = Risk Neutral Nash Equilibrium

## Introduction

This thesis is a collection of three self-contained chapters about first-price auctions with independent private values.

Bidding above the risk neutral Nash equilibrium (overbidding) has been a consistent finding in the experimental literature of first-price private value auction for decades. Risk aversion was initially widely accepted as the explanation of such a phenomenon. Recently, some papers, such as Lange and Ratan (2010) and Delgado, Schotter, Ozbay, and Phelps (2008), have identified that loss aversion is also responsible for overbidding.

Chapter 1 is therefore inspired by this: if loss aversion can play a role in overbidding in the standard first-price auction where the losers actually have no monetary loss, then what happens if we impose a device in which the losers face a ‘loss’? We conjecture that the overbidding should be even stronger, such that revenue increases.

We design a novel device called a ‘payback scheme’, in which all the bidders receive an initial capital balance before the auction starts, and they can use any of it to submit bids. However, only the winner keeps the initial capital balance, whereas all the losers have to ‘pay it back’. We provide and compare the homogeneous risk aversion and loss aversion equilibrium bidding models, and the corresponding revenue predictions for a first-price private value auction in a simple single-unit auction scenario with uniformly distributed private values. The model predicts that the scheme can increase the seller’s revenue if the bidders are loss averse and the more loss averse the bidders, the more revenue should increase.

By conducting a series of experiments, we compare the realised bids and the revenues with and without the payback scheme. We find that, even though the bidders on average are loss averse, whether the payback scheme generates more revenue depends on: i) the

market size and ii) the amount of the initial capital balance retained by the winner relative to the maximum private value. Moreover, we also identify that for each bidder, the elicited risk preferences are not stable across different institutions – a first-price auction method and a Becker-DeGroot-Marschask (BDM) lottery procedure.

As mentioned above, risk aversion is a commonly used explanation for the overbidding phenomenon in first-price private value auctions. Originally, the literature assumed bidders displayed a homogeneous risk aversion attitude (Holt, 1980; Maskin & Riley, 1980). Cox, Roberson, and Smith (CRS) (1982), and Cox, Smith and Walker (CSW) (1988) develop a general constant relative risk aversion model (CRRAM) that incorporates three cases for the bidders:

- They are all risk neutral.
- They are equally risk averse.
- They differ in their risk averse attitudes.

The CRRAM relaxes the unrealistic assumption that the bidders' risk attitudes are the same, and instead assumes that the bidders' risk parameters can differ and are from a commonly known distribution. However, such a model is only valid to explain the bids that do not exceed a certain limitation. Beyond this limitation, the bid-private value relationship becomes nonlinear. The exact value of this limitation directly depends on the bidders' prior belief of the support of the risk parameter distribution.

Henceforth, two questions are brought about. Firstly, how does changing the assumption of the distribution influence the nonlinear part of the bid functions? Secondly, how does changing the support of the distribution influence the estimated individual risk parameter? Van Boening, Rassenti and Smith (1998) solve the first question. Chapter 2 addresses the second issue by re-estimating the datasets from three papers by Cox, Roberson, and Smith

(1982), and Cox, Smith, and Walker (1983, 1985), which include various market sizes – 3, 4, 5, 6, and 9. We consider two representative cases of the distribution support for the risk parameters: risk neutral ( $r_{max} = 1$ ) and risk loving ( $r_{max} = 2$ ).

We draw two main conclusions from the analysis. First, changing the support of the distribution indeed varies the estimated individual risk parameter. Secondly, and more importantly, despite the first finding, the two distributions themselves remain unchanged.

In addition, we also compare individuals' loss aversion coefficients elicited from our lottery experiment (which also appears in Chapter 1) with another lottery experiment conducted in 2014 by Pezanis-Christou and Wu. Similar to the analysis of risk parameter distributions above, we find that the distributions of the average loss aversion parameters are stochastically equivalent for the two experiments.

The first two chapters both study the first-price auction in the single-unit application. Chapter 3 transfers the research object to the multi-unit first-price auction, more specifically, sequential first-price private value auctions. Building on the results of Chapter 1, we compare how bidders respond to 'loss' and how that would influence the seller's revenue across the single-unit and sequential auction institutions.

To achieve this, we re-examine the data of Keser and Olson (1996). They have a particularly interesting penalty treatment, in which all the bidders act as agents, and those who fail to acquire an item (losers) need to pay a penalty to the implicit principal. Such a treatment has a connection with the payback scheme in Chapter 1 since in the two treatments, the losers both face a monetary loss. Therefore, it allows us to have a full picture of how bidders respond to 'loss' and how that would influence the seller's revenue in both the single-unit and the sequential auctions contexts.

We rigorously derive the risk neutral Nash equilibrium (RNNE) bidding strategy and consequential revenue prediction for each stage in the sequential first-price private value auction with a penalty. Overall, we find that the penalty influences bidding behaviour differently from the RNNE bidding strategy, in the following four aspects: i) the estimated piecewise linear bidding function is non-monotonic; ii) the price declines across stages instead of remaining constant; iii) the bids actually depend on the price of the previous stage; iv) the allocation efficiency is not 100% for each stage.

## Chapter 1: Payback scheme in first-price private value auctions: an experimental study

### 1.1 Introduction

First-price sealed bid auction, as one of the four primary auction types (the other three are English, Dutch, and second-price auction) used to allocate items, is widely adopted in the field. The bidding rule is easily understood: The bidders write their bids for the item and deliver them to the auctioneer; the auctioneer determines the highest bidder, and the highest bidder gets the item for a price equal to his own bid. There are two forms of application for first-price auctions. The first is that the bidders are ‘buyers’, and the highest bidder wins the auction; the other is the bidders are ‘suppliers’ (i.e. construction contracts as in Vickrey, 1961), and the lowest bidder wins. In this chapter, we focus on analysing the first-price sealed bid auction with independent private value bidders. For independent private value (IPV) auctions, each bidder knows the value of the item to himself and the distribution from which the bidders’ valuations are independently drawn.

Vickrey (1961) was the first to apply game theory to build the theoretical model for independent private value actions. By assuming risk neutral bidders, he derived the unique risk neutral Nash equilibrium (RNNE) bid functions for first-price and second-price auctions given that the private values are drawn from a uniform distribution. Furthermore, he demonstrated that the first-price auction is strategically equivalent to the Dutch auction, and pointed out that the second-price auction (Vickrey auction) is equivalent to the conventional English auction.

However, overbidding in first-price auctions with independent private values is consistent with experimental findings which suggest that bidders consistently bid above the RNNE prediction (CSW, 1982; Kagel & Roth, 1995; Kagel & Levin, 2011). This overbidding

anomaly was initially explained by the constant relative risk aversion model - CRRA (CSW, 1988). The intuition behind this is that the subject prefers a sure gain by submitting a higher bid to a risky but potentially greater gain with a lower bid. However, as mentioned by Kagel and Roth (1995, p. 525), ‘risk aversion is one element, but far from the only element generating bidding above the RNNE’, many alternative behavioural models also give explanations for this anomaly.

Goeree, Holt and Palfrey (2002) compare bidders’ behaviour with a two-bidder market in two first-price private value auction treatments (low and high private values with a group of six discrete values in each). The treatments have the same RNNE bid, but differ in the curvature of the loss function. Overbidding is observed for both treatments and is more common in the high value treatment as conjectured. They find that the quantal response equilibrium (QRE) model with risk aversion fits the bidding data well, whereas the ‘pure joy of winning’ model is reasonable, but does significantly worse.<sup>1</sup>

Dorsey and Razzolini (2003) study the bidding behaviour in two equivalent environments: the first-price private value auction, and the lottery choice. In the auction experiment, each bidder competes against three simulated bidders who use the RNNE bidding strategy. With regards to the first-price auction, there are two treatments - a baseline treatment and one in which each individual is provided the probability of winning with a particular bid, after which he can either submit or revise the bid. By examining the bidding behaviour, they find that showing the subjects the probability of winning the auction causes the bids at high private values to become less aggressive and closer to the RNNE bids, thus suggesting that the misperception of the probabilities of winning plays some role in overbidding.

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<sup>1</sup> Cox, Smith, and Walker (1988) also incorporate ‘Joy of winning’ in their CRRA model.

Filiz and Ozbay (2007) introduce regret theory, which incorporates the payoffs from the forgone alternatives in the expected utility function to explain overbidding. The study implements a series of one-shot first-price auction experiments in order to analyse the impact of anticipated loser and winner regret in first-price auctions using a between-subject design. Choosing a one-shot game instead of the typical repeated rounds game rules out the learning effect. There are three treatments based on what information is revealed to all subjects at the end of the auction - that is, the winning bid (loser regret), the second highest bid (winner regret), and no information feedback. They find that subjects do not seem to anticipate winner regret, as the estimated slope of the bid function (0.77) is not significantly different from that in the no feedback treatment (0.79), whereas they do identify anticipated loser regret, as the estimated slope of the bid function is significantly higher under this condition (0.87).

So far, the explanations discussed are all based on the expected utility framework. Another strand of literature considers the endogenous reference dependence (introduced by Koszegi & Rabin, 2006) to analyse standard auctions, such as Lange and Ratan (2010). They develop the Koszegi-Rabin framework in first- and second-price auctions and find an additional explanation - loss aversion also leads to overbidding in induced private value first-price auctions.<sup>2</sup> For the standard first-price auction, there is no monetary loss for the bidders since the payoff for the losers is zero; they fail to buy the item but also do

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<sup>2</sup> However, loss aversion does not lead to overbidding in the field auction setting. This is because field auctions also include a commodity dimension whereas the lab auctions only include one dimension. More specifically, in the lab auctions, normally the auction item and the bidders' payments are measured in the same monetary dimension (the property of inducing value for the auction item permits that we can perfectly observe the private values) whereas in the field auctions, the auction item (i.e. wine, house, etc.) is in the commodity dimension. Intuitively, in the lab (induced-value) auctions, the loss (relative to the expected payoff) occurs when a subject loses the auction and increasing bid would decrease the potential loss of money. To the contrary, in the field (commodity) auctions, the monetary loss happens when a subject wins the auction and decreasing bid would decrease the potential loss of money. Overall, the effect of loss aversion in the money dimension differs in the lab and field first-price auctions.



not pay at all. As a result, the ‘loss’ actually occurs when the bidder expects to win but loses the auction. Naturally, we consider what might happen if we come up with an auction scheme in the first-price auction in which the losers really lose some money. Would such a scheme generate even stronger overbidding? If so, we also want to know whether it will enhance the seller’s revenue, since maximising such revenue is one major goal of an auction design.

Therefore, we come up with a new and simple device which permits us to test the above conjectures. This device is called ‘payback’, in which each bidder receives an initial capital balance before the auction starts and can use the money when submitting his bid. However, after the highest bid has been announced, only the winner can keep the initial capital balance whereas all the losers need to ‘pay back’ the initial capital balance to the seller. Thus, within this scheme, we stimulate the losers facing a ‘loss’ relative to the situation in which they receive the initial capital balance.

Loss aversion would arguably play a role in this scheme. Kahneman and Tversky (1979) first formulated the concept of loss aversion which before was widely argued in psychology. A central result of loss aversion is that the people are much more sensitive to potential losses than potential gains. The phenomenon of loss aversion is well established in the experimental literature, and it is widely observed in both risky and riskless choice decisions (Rabin, 2000; Fehr & Goette, 2007; Kahneman, Knetsch, & Thaler, 1990).

Much of the research relevant to loss aversion also lies within neuroeconomics (Tom et al., 2007; Delgado et al., 2008). Anticipated or actual losses may cause individuals to experience negative emotions leading to loss aversion. A joint paper by cognitive neuroscientists and economists (Delgado et al., 2008) is closely relevant to this study.

This novel paper provides insight into the neural circuitry of experimental auctions and uses such insight to understand overbidding. They design three treatments: baseline, ‘loss-frame’ which emphasises loss, and ‘bonus-frame’, which emphasises bonus (or gain). Overall, they find a stronger tendency to overbid in the ‘loss-frame’ treatment. This chapter exploits this stronger tendency to overbid in ‘loss-frame’ auctions to potentially increase the seller’s revenue. We also provide the Nash equilibrium bidding strategies under two assumptions for bidders: risk aversion and loss aversion. This allows us to obtain a hypothesis that the seller’s revenue should be increased with the payback scheme if the subjects are loss averse.

In this chapter, we conduct a series of first-price private value auctions with and without the payback scheme using a within-subject design, thus eliminating the subject-specific effect. In addition, both a large market ( $n=6$ ) and a small market ( $n=3$ ) are chosen to compare the corresponding bidding behaviour and the revenue results.

Our study and Delgado et al. (2008) both use the same measurement for the seller’s revenue: the winner’s bid minus the initial capital balance given to him. However, the main experiment result is different. In Delgado et al. (2008)’s experiment, both the bids and the revenue are greater in the ‘loss-frame’ treatment relative to the ‘baseline’ treatment. To the contrary, in our payback scheme treatment, even though the subjects indeed bid higher, actually the seller’s revenue is significantly less than in the standard first-price auction for the 6-bidder market and not significantly different for the 3-bidder market. Therefore, we conclude that using the payback scheme to enhance revenue depends vitally on the amount of the initial capital balance relative to the maximum possible private value. In Delgado et al. (2008), the ratio is 15%, whereas such a ratio increases to 50% in our experiment. At such a high ratio, the induced increase in bids

cannot offset the cost of the initial capital balance retained by the winner, which leads to the payback scheme failing to increase revenue in our experiment.

The remainder of the chapter is laid out as follows. In the next section we introduce the theoretical framework and the predictions of the Nash equilibrium bids and expected revenues. In Section 1.3 we present our experimental design in detail. Section 1.4 and 1.5 report the main results. In Section 1.6, we compare the risk aversion coefficients across different institutions, and then in Section 1.7 we explore the conditions when the payback scheme works in terms of enhancing revenue. More specifically, we re-estimate the bid function using the experimental data provided by Delgado et al. (2008) and compare such results with our experiment. Finally, Section 1.8 concludes this chapter.

## 1.2 Theoretical models

### 1.2.1 Preliminaries

In this section we derive bidders' equilibrium bidding strategies in a payback scheme first-price auction. Consider there are  $n$  bidders participating in a first-price sealed-bid auction. They compete for a single object and submit sealed bids  $b_1, b_2, \dots, b_n$ . The bidder who submits the highest bid is awarded the object, and pays his bid. Each bidder  $i = \{1, 2, \dots, n\}$  has a private value  $v_i$  which is an independent draw from a uniform distribution  $F$  defined on  $[0, 1]$ . The number of bidders  $n$  and the distribution  $F$  are common knowledge, but the value realization  $v_i$  is private information.

With the payback scheme, each bidder receives an initial capital balance  $K$  before the auction starts, and he could use any proportion of  $K$  to submit his bid. However, he keeps

the money  $K$  only if he is the winner; if he loses the auction, he has to give the money  $K$  back to the seller. That is the reason why we name such a scheme ‘payback’.

We derive the equilibrium bidding strategies by considering the signalling problem of bidder  $i$ , given that all other bidders ( $j \neq i$ ) use the same increasing, differentiable bidding strategy  $b(\cdot)$  to map their own private values into bids. Bidder  $i$  is not obliged to reveal his true type  $v_i$ , so he can select a private value  $z_i$  from the uniform distribution  $F$  and submit a bid of  $b(z_i)$ . Next we use the revelation principle to derive the symmetric Nash equilibrium bidding strategy. More specifically, we verify bidder  $i$  has no incentive to bid as if he had a private value  $z_i \neq v_i$ .

### 1.2.2 Risk Averse Symmetric Nash Equilibrium model (RASNE)

Vickrey (1961) was the first to derive the Nash equilibrium bidding function in independent private-value auctions assuming that bidders are all risk neutral. Holt (1980), Maskin and Riley (1980), and Harris and Raviv (1981) extend the Vickrey model to the case that bidders are risk averse. More specifically, they assume that the bidders display a homogeneous risk averse attitude and the corresponding expected revenue is greater than if they were risk neutral.

Since the assumption of the bidders sharing the same risk attitude is restrictive, Cox, Roberson, and Smith (1982) construct an equilibrium bidding model (CRRA) that permits bidders to differ in their risk attitudes with a utility function  $u_i(y) = y^{r_i}$  where the individual constant relative risk preference parameter  $r_i$  is from a probability distribution  $\Phi$  on  $[0, 1]$ . Each bidder knows his own risk parameter  $r_i$  as well as the probability distribution  $\Phi$ . An important feature of the bid function  $b_i = \frac{n-1}{n-1+r_i} v_i$  is that it only

applies to bids that do not exceed  $\bar{b} = \frac{n-1}{n}$  which is the maximum bid that the least risk averse (in other words, risk neutral) bidder would submit.

Cox, Smith, and Walker (1988) generalise the CRRA model to  $r_i \in (0, r_{max}]$ , where  $r_{max} \geq 1$  which stands for the risk parameter for the least risk averse bidder. In this model, the least risk averse bidder could be a risk neutral or a risk-loving bidder, which depends on the prior belief of  $r_{max}$ . We will discuss how changing  $r_{max}$  influences the estimated individual risk parameter in Chapter 2. In this chapter, we focus our analysis on the Nash equilibrium bidding strategy in a first-price payback scheme auction for homogeneous bidders.

The probability of bidder  $i$  (bidding as if he had a private value  $z_i$ ) winning the auction is that all the other  $n - 1$  bidders' private values are smaller than  $z_i$ , which is  $F(z_i)^{n-1} = z_i^{n-1}$ . Bidder  $i$ 's expected utility is defined as

$$\begin{aligned} EU_i(z_i) &= z_i^{n-1}(K + v_i - b(z_i))^r + (1 - z_i^{n-1})(K - K) \\ EU_i(z_i) &= z_i^{n-1}(K + v_i - b(z_i))^r \end{aligned} \tag{1.2.1}$$

It must have the property that for any true private value  $v_i$ , the expected utility function (1.2.1) is maximised by setting  $z_i = v_i$ . Therefore,  $v_i$  should satisfy the below first order condition

$$\frac{\partial EU_i(z_i)}{\partial z_i} \Big|_{z_i=v_i} = 0 \tag{1.2.2}$$

Which yields the following first order differential equation

$$b'(v_i) = \frac{(n-1)(K + v_i - b(v_i))}{v_i r} \tag{1.2.3}$$

for all  $v_i$  in the interval  $[0,1]$ , equation (1.2.3) is solved by the following risk averse symmetric Nash equilibrium (RASNE) bidding function:<sup>3</sup>

$$b(v_i)^{RASNE} = K + \frac{n-1}{n+r-1} v_i \quad (1.2.4)$$

We substitute equation (1.2.4) in the second order condition  $\frac{\partial^2 EU_i(z_i)}{\partial z_i^2} \big|_{z_i=v_i} = -\frac{(n-1)}{r \cdot v_i} < 0$  which satisfies the maximising profit requirement. Therefore, if every bidder is using the same bidding function  $b(\cdot)$ , it is optimal for all bidders to reveal their true types.

When  $r = 1$  then equation (1.2.4) reverts to Vickrey's benchmark risk neutral Nash equilibrium (RNNE) model

$$b(v_i)^{RNNE} = K + \frac{n-1}{n} v_i \quad (1.2.5)$$

### 1.2.3 Loss Averse Symmetric Nash Equilibrium model (LASNE)

In this section, instead of assuming subjects display a homogeneous risk averse attitude, we presume that they share a homogeneous loss aversion coefficient  $\lambda > 0$ . Such a coefficient only plays a role when subjects experience a loss. A subject with  $\lambda > 1$  is loss averse, and the greater the value of  $\lambda$ , the more loss averse the subjects is. A subject with  $\lambda = 1$  is loss neutral, whereas  $\lambda < 1$  indicates the subject is gain-seeking. To simplify the model, we also assume the subjects are risk neutral where  $r = 1$ . Therefore, bidder  $i$ 's expected utility is defined as

$$EU_i(z_i) = z_i^{n-1} (K + v_i - b(z_i)) + (1 - z_i^{n-1}) (K - \lambda K) \quad (1.2.6)$$

---

<sup>3</sup> The full derivation of  $b(v_i)^{RASNE}$  is in Appendix A.

As in the last section, for any private value  $v_i$ , the expected utility function (1.2.6) is maximised by setting  $z_i = v_i$ . Therefore  $v_i$ , should again, satisfy

$$\frac{\partial EU_i(z_i)}{\partial z_i} \Big|_{z_i=v_i} = 0 \quad (1.2.7)$$

Hence, we obtain the following first order differential equation

$$b'(v_i) = \frac{(n-1)(\lambda K + v_i - b(v_i))}{v_i} \quad (1.2.8)$$

for all  $v_i$  in the interval  $[0,1]$ , equation (1.2.8) is solved by the following loss averse symmetric Nash equilibrium (LASNE) bidding function:

$$b(v_i)^{LASNE} = \lambda K + \frac{n-1}{n} v_i \quad (1.2.9)$$

When subjects are loss neutral (where  $\lambda = 1$ ), then equation (1.2.9) also reverts to Vickrey's benchmark risk neutral Nash equilibrium (RNNE) model as in Equation (1.2.5).

#### 1.2.4 Expected revenue predictions

In equilibrium, the seller's expected revenue is determined by evaluating the corresponding Nash equilibrium bidding strategy at the expected highest value in the uniform distribution  $[0, 1]$ , which is  $\frac{n}{n+1}$ . Hence, with regards to the RASNE model

$$\begin{aligned} ER^{RASNE} &= K + \frac{n-1}{n+r-1} \times \frac{n}{n+1} - K \\ ER^{RASNE} &= \frac{n(n-1)}{(n+r-1)(n+1)} \end{aligned} \quad (1.2.10)$$

When  $r = 1$  then equation (1.2.10) becomes

$$ER^{RNNE} = \frac{n-1}{n+1} \quad (1.2.11)$$

with respect to the LASNE model, the seller's expected revenue is equal to

$$ER^{LASNE} = \lambda K + \frac{n-1}{n} \times \frac{n}{n+1} - K$$

$$ER^{LASNE} = (\lambda - 1)K + \frac{n-1}{n+1} \quad (1.2.12)$$

When  $\lambda = 1$  then equation (1.2.12) also becomes equation (1.2.11), from which we obtain that the expected revenue  $ER_{k=0}^{n=6} = ER_{k=0.5}^{n=6} = 0.71$  and  $ER_{k=0}^{n=3} = ER_{k=0.5}^{n=3} = 0.50$ .

In addition, the predictions allow us to formulate the following hypotheses:

**Hypothesis 1a** (RASNE): The seller's expected revenue would not be influenced by the payback scheme if subjects are risk averse.

$$R_{k=0} = R_{k=0.5}$$

**Hypothesis 1b** (LASNE): The seller's expected revenue would be enhanced with the payback scheme if subjects are loss averse.

$$R_{k=0.5} \geq R_{k=0} \text{ (if } \lambda \geq 1 \text{)}$$

**Hypothesis 2:** (RASNE & LASNE): The seller's expected revenue would always be greater in the larger market.

$$R_{n=6} > R_{n=3}$$

### 1.3 Experimental design

We ran the experiments using the software Z-Tree at the University of Adelaide's 'Adelaide Laboratory for Experimental Economics' (Adlab) in April of 2016. Sixty



subjects from the undergraduate and postgraduate population of the University were recruited by the ORSEE system and participated in 4 sessions. In a given session, each subject participated in 4 experiment stages: two lottery experiments and two auction experiments. Quiz questions were given to subjects before each experiment stage, and a stage only began when all subjects answered the quiz questions correctly. Each session lasted about 90 minutes. Subjects received written instructions which were read aloud and could ask questions to the experimenter in private. A copy of the experimental instructions is given in Appendix A. Including a show-up fee of \$10, subjects earned \$20 on average.

The result of each experiment was not revealed until the end of the session, in order to keep the decision for each experiment task independent. The subject was paid according to his aggregate payoffs from the 4 experiment stages.

### **1.3.1 The first lottery experiment stage**

The aim of the first lottery experiment is to measure subjects' loss aversion attitudes. Subjects decided whether or not to accept 14 risky lotteries as shown in the first column of Table 1.3.1, one of which would be randomly selected for payment. For each lottery, there is a 50% chance of winning and a 50% chance of losing.

To determine which lottery would be chosen, a number between 1 and 14 was randomly drawn for each subject. If the subject chose to 'Accept' the corresponding lottery, his final payoff was adjusted according to the result of the lottery; if the subject chose to 'Reject', then he got zero from this experiment task. As explained earlier, in order to keep the decision for each experiment task independent, the result of this first lottery experiment was not revealed to the subject until the end of the session.

Table 1.3.1 The design of the 14 risky lotteries in the first lottery stage

|     | Lottery 50%/50% chance  | Expected Value |
|-----|-------------------------|----------------|
| #1  | Lose \$0.5 or win \$9.5 | \$4.50         |
| #2  | Lose \$1 or win \$9     | \$4            |
| #3  | Lose \$1.5 or win \$8.5 | \$3.50         |
| #4  | Lose \$2 or win \$8     | \$3            |
| #5  | Lose \$2.5 or win \$7.5 | \$2.50         |
| #6  | Lose \$3 or win \$7     | \$2            |
| #7  | Lose \$3.5 or win \$6.5 | \$1.50         |
| #8  | Lose \$4 or win \$6     | \$1            |
| #9  | Lose \$4.5 or win \$5.5 | \$0.50         |
| #10 | Lose \$5 or win \$5     | \$0            |
| #11 | Lose \$5.5 or win \$4.5 | -\$0.5         |
| #12 | Lose \$6 or win \$4     | -\$1           |
| #13 | Lose \$6.5 or win \$3.5 | -\$1.5         |
| #14 | Lose \$7 or win \$3     | -\$2           |

### 1.3.2 The second lottery experiment stage

Following the completion of the first lottery task, a second lottery experiment was conducted for eliciting subjects' certainty equivalents for 11 lotteries.<sup>4</sup> Each lottery, initially owned by the subject, has a 50% chance of a high payoff  $H$  and a 50% chance of a low payoff  $L$  as shown in columns 1-3 of Table 1.3.2. Certainty equivalents were elicited using the Becker-DeGroot-Marschak (BDM) (1963) incentive mechanism, which gives the subject an incentive to report his true valuations for the corresponding lotteries. The procedure was as follows. The subject was asked to state a minimum selling price  $p_s$  (between the high and low payoff) for each lottery, with the knowledge that a random buying price  $p_b$  (also between the high and low payoff) would be drawn to determine if the lottery would be sold to the computer. If  $p_b \geq p_s$ , the subject received the randomly drawn buying price; otherwise, he received the outcome of the lottery.

<sup>4</sup> We used the same 11 lotteries as in Kocher, Pahlke, and Trautmann (2010). The differences between the high and low payoffs for the lotteries are always even numbers: 2, 4, 6, 8, and 10. It makes the arithmetic easier for subjects.

As with the first lottery experiment, only one lottery would be chosen for each subject to decide his payoff in this BDM lottery task, and the result would not be revealed until the end of the session.

Table 1.3.2 The design of the 11 lotteries in the second lottery stage

| Lottery | High payoff<br>(50% chance) | Low payoff<br>(50% chance) | Expected<br>value |
|---------|-----------------------------|----------------------------|-------------------|
| #1      | 12.76                       | 4.76                       | 8.76              |
| #2      | 8.30                        | 2.30                       | 5.30              |
| #3      | 10.70                       | 2.70                       | 6.70              |
| #4      | 6.52                        | 2.52                       | 4.52              |
| #5      | 13.22                       | 5.22                       | 9.22              |
| #6      | 8.06                        | 2.06                       | 5.06              |
| #7      | 6.36                        | 2.36                       | 4.36              |
| #8      | 13.20                       | 3.20                       | 8.20              |
| #9      | 9.76                        | 5.76                       | 7.76              |
| #10     | 12.76                       | 6.76                       | 9.76              |
| #11     | 8.01                        | 2.01                       | 5.01              |

*Note: Numbers in columns 2-4 show amounts in AUD. The expected value for the corresponding lottery was not shown to the subjects.*

### 1.3.3 The auction experiment stages

The auction experiment was designed to test whether the payback scheme enhances the seller's revenue. We used within-subject variation. Therefore, subjects were exposed to two treatments: standard first-price auction, and payback scheme first-price auction,

which we refer to as k0 and k5 treatment hereafter. In both auction stages, subjects were in the same group of six bidders for 20 rounds. The k0 treatment is the control treatment since it accords with a large number of laboratory studies.

In this chapter, the k5 treatment is the novel treatment. In the k5 treatment, subjects received \$5 as the initial capital balance they could use to bid before each auction started. Only the winner got to keep the \$5; all the losers had to pay the \$5 back. Due to the order effect that exists in the within-subject design, it was necessary to run the treatments in both orders: k0k5 order and the reverse, k5k0 order. In the k0k5 order of the treatment, subjects participated in the standard private value first-price auction for the first 20 rounds and then for the second 20 rounds, they were switched to the conditions of the payback scheme. With respect to the k5k0 order, subjects were exposed to the treatments in reversed order.

To study the effect of the payback scheme on seller's revenue, we also examined two different market sizes: 6-bidder market and a 3-bidder market using a between-subject design. For both market sizes, at the beginning of the auction stage, the computer randomly allocated subjects to markets of size  $n = 6$ . Additionally, to form the 3-bidder market, in each auction round, the fixed group of six bidders was re-matched into two 3-bidder markets.<sup>5</sup> This matching method, on the one hand, provides independent units of observation. On the other hand, it constitutes a comparison with the 6-bidder market. We use the notation k0k5\_6 to represent the experiment session with the k0k5 order in a 6-bidder market.

In each round, subjects' private values were independently drawn from a uniform distribution defined on  $[\$0, \$10]$ . Each subject was required to submit a bid equal to or

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<sup>5</sup> Such a design is similar to Schram and Onderstal (2009), except that in their experiment, the subjects did not know the 3-bidder market was formed within the fixed group of six bidders.

below his private value in the k0 treatment, and equal to or below his private value plus \$5 in the k5 treatment. The winner was the subject who submitted the highest bid and paid a price equal to his bid. In the case of a tie, the winner was randomly chosen among the bidders who submitted the highest bid. In each market, at the end of each auction round, the winner's bid (but not identity) was disclosed to all the subjects. Table 1.3.3 summarises our auction experiments.

Each subject's payoff in the two auction stages was decided by the computer, which randomly chose two auction rounds for each treatment. The summation of the payoffs from the four rounds was the subject's payoff from the auction experiment.

Table 1.3.3 The design of the auction experiments

| Market size | Session | Treatment                         | # Subjects | # Groups |
|-------------|---------|-----------------------------------|------------|----------|
| 6           | k0k5_6  | K0 <sub>1</sub> , K5 <sub>2</sub> | 18         | 3        |
|             | k5k0_6  | K5 <sub>1</sub> , K0 <sub>2</sub> | 18         | 3        |
| 3           | k0k5_3  | K0 <sub>1</sub> , K5 <sub>2</sub> | 12         | 2        |
|             | k5k0_3  | K5 <sub>1</sub> , K0 <sub>2</sub> | 12         | 2        |

## 1.4 Descriptive analysis for two lottery tasks

### 1.4.1 Loss aversion

As in Rabin (2000) and Fehr and Goette (2007), the rejection of a small-stake risky lottery with a positive expected value can be interpreted as loss aversion instead of risk aversion. So we can use the first lottery task to measure the subject's loss aversion attitude. In this task, the least loss-averse (i.e. the most gain-seeking)<sup>6</sup> subject would choose to accept all 14 lotteries because of the 50% chance of winning some money, even though the expected values are negative from lottery #11 to lottery #14. To the contrary, an extremely loss-

<sup>6</sup> Here, we adopt the terminology 'gain-seeking' as per Abdellaoui, Bleichrodt and L'Haridon (2008).

averse subject would choose to reject all the 14 lotteries since all the lotteries include a 50% chance of losing money. Overall, a subject would reject more lotteries if he is more loss-averse. Hence, we can use a subject's switch point from accepting to rejecting a specific lottery to measure his loss aversion. Among all the 60 subjects, four subjects have more than one switch point (6.67% of all the subjects). For these subjects, we only analyse the first switch point as per Prasad and Salmon (2013).<sup>7</sup>

Before devising a framework to calculate each subject's loss aversion coefficient, it is necessary to first have a general idea about the distribution of accepted lotteries among all the 60 subjects. Figure 1.4.1 shows the distribution of the number of accepted lotteries in the loss aversion measurement stage. The mode of the number of accepted lotteries is 6 (12 subjects), in which the expected value is \$2.

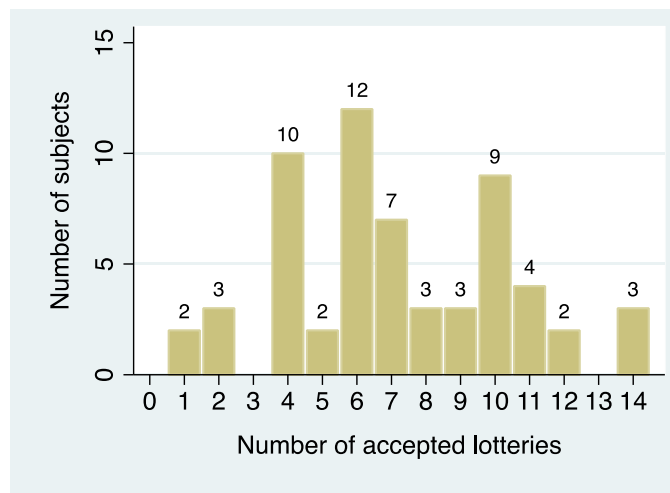


Figure 1.4.1 Distribution of the number of accepted lotteries

Suppose the lottery chosen to determine the subject's payoff is (50% chance of winning  $w$ , 50% chance of losing  $w'$ ). We adopt the expected utility framework to illustrate the utility a subject gets from the first lottery stage:

<sup>7</sup> Some papers, like Laury and Holt (2005), only investigate the 'one switch point' choice pattern and ignore those subjects with more than one switch point.

$$u(w) = \begin{cases} w, & w \geq 0 \\ \lambda w', & w' < 0 \end{cases}$$

where  $\lambda$  is the loss aversion coefficient. The first equation represents the utility of a subject winning whereas the second equation measures the disutility of losing. A larger loss aversion coefficient  $\lambda$  represents that the subject is more loss-averse, as the feeling of losing money is more painful. A subject will accept a lottery if:

$$prob(gain)u(w) + prob(lose)u(w') > 0$$

A subject will reject a lottery if:

$$prob(gain)u(w) + prob(lose)u(w') < 0$$

When a subject is indifferent between accepting and rejecting a lottery, it must be that:

$$prob(gain)u(w) + prob(lose)u(w') = 0$$

$$prob(gain) = prob(lose) = 50\%$$

$$u(w) + u(w') = 0$$

$$w + \lambda w' = 0$$

$$\lambda = -\frac{w}{w'}$$

The above equation is satisfied when we exactly know the lottery for which the subject is indifferent between accepting and rejecting. However, we can only observe a switch point for each subject. For instance, if a subject accepts the first 2 lotteries, but rejects the next 12 lotteries, we know the accurate indifferent lottery must lie between #2 and #3. Therefore, according to this model, the loss aversion coefficient  $\lambda$  must lie in the interval (5.67, 9]. As mentioned by Anderson and Mellor (2009), a common technique for dealing with this estimation problem is to use an interval regression model. The below model

accounts for interval censoring of the dependent variable, in this scenario loss aversion coefficient  $\lambda$ , as well as left and right censoring.<sup>8</sup> The subjects who accept between 1 and 13 lotteries are interval censored observations; those who accept all the 14 lotteries are left censored observations;<sup>9</sup> as for the subjects who reject all the lotteries, they are right censored observations.

$$\lambda_i^* = \mu + \varepsilon_i \quad (1.4.1)$$

We do not observe subject  $i$ 's loss-averse attitude  $\lambda_i$  ( $\lambda_i > 0$ ) directly. However, we instead observe  $y_i$ , which indicates the number of lotteries that subject  $i$  accepts. The notation  $y_i$  implies a range for  $\lambda_i^*$ , which is delimited by  $[\lambda_{min}, \lambda_{max}]$ . For this reason, instead of  $\lambda_i$ , we model the latent variable  $\lambda_i^*$  as in equation (1.4.1). In equation (1.4.1),  $\varepsilon_i$  is a normally distributed error with mean zero and variance  $\sigma^2$ .

A maximum likelihood procedure has been used to estimate this model. After obtaining the estimated intercept  $\mu$ , subject  $i$ 's expected loss aversion coefficient given the corresponding number of accepted lotteries is computed in the following way:

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<sup>8</sup> Interval censoring describes the case when a data point is somewhere on an interval between two values. Left (right) censoring represents that when a data point is below (above) a certain value but it is unknown by how much.

<sup>9</sup> However, we know that the loss aversion coefficient must be a positive figure. So for those who accept all the 14 lotteries are still interval censored observations.



$$\begin{aligned}
E[\lambda_i^* | y_i = 0] &= E(\lambda^*) \frac{\Phi(-\sigma + b)}{\Phi(b)} \\
E[\lambda_i^* | y_i \in \{1, \dots, 13\}] &= E(\lambda^*) \frac{\Phi(\sigma - a) - \Phi(\sigma - b)}{\Phi(b) - \Phi(a)} \\
E[\lambda_i^* | y_i = 14] &= E(\lambda^*) \frac{\Phi(\sigma - a)}{\Phi(-a)} \\
E(\lambda^*) &= \exp\left(\mu + \frac{\sigma^2}{2}\right) \\
a &= \frac{\lambda_{\min} - \mu}{\sigma} \\
b &= \frac{\lambda_{\max} - \mu}{\sigma}
\end{aligned}$$

The range of  $[\lambda_{\min}, \lambda_{\max}]$  for the corresponding number of accepted lotteries and the expected loss aversion coefficient  $\lambda^*$  are shown in Table 1.4.1. We also report the related percentage of subjects for each number of accepted lotteries. Within all the 60 subjects, 15% of them accept all ten lotteries with a non-negative expected value, a further 15% of subjects accept at least one lottery with a negative expected value, and the remaining 70% of subjects reject at least lottery #10 (which has an expected value of zero) or some lotteries even with positive expected values. The median subject accepts lotteries #1 to #7, which implies that the median value of  $\lambda$  is 1.68.<sup>10</sup> Such a result is qualitatively similar to the median value of  $\lambda$  (2.25) reported by Tversky and Kahneman (1992). Hence, we find that loss aversion is a significant pattern for the subjects.

It is instructive to compare the results with those of a similar experiment. The paper by Gächter, Johnson, and Herrmann (2007) measure the individual-level loss aversion using six 50-50 lotteries. The winning money is fixed at €6, whereas the loss varies from €2 to

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<sup>10</sup> We can think of an example to have a intuitively understanding of  $\lambda = 1.68$ . That is, a subject must gain \$1.68 to offset the disutility of losing \$1. Therefore, as  $\lambda$  increases, a subject need to gain more money to compensate the disutility of losing \$1. The subject is more loss averse, so to speak.

€7. They find a similar result that the median subject has a loss aversion coefficient  $\lambda = 1.2$ .

Table 1.4.1 The loss aversion parameter for the corresponding number of accepted lotteries

| # Accepted Lotteries | # Subjects | Percentage (%) | Cum. Percentage (%) | $[\lambda_{min}, \lambda_{max}]$ | $\lambda$ |
|----------------------|------------|----------------|---------------------|----------------------------------|-----------|
| 0                    | 0          | 0.00           | 0.00                | $(19, \infty)$                   | n.a.      |
| 1                    | 2          | 3.33           | 3.33                | $(9, 19]$                        | 9.49      |
| 2                    | 3          | 5.00           | 8.33                | $(5.67, 9]$                      | 6.44      |
| 3                    | 0          | 0.00           | 8.33                | $(4, 5.67]$                      | n.a.      |
| 4                    | 10         | 16.67          | 25.00               | $(3, 4]$                         | 3.47      |
| 5                    | 2          | 3.33           | 28.33               | $(2.33, 3]$                      | 2.66      |
| 6                    | 12         | 20.00          | 48.33               | $(1.86, 2.33]$                   | 2.10      |
| 7                    | 7          | 11.67          | 60.00               | $(1.5, 1.86]$                    | 1.68      |
| 8                    | 3          | 5.00           | 65.00               | $(1.22, 1.5]$                    | 1.36      |
| 9                    | 3          | 5.00           | 70.00               | $(1, 1.22]$                      | 1.11      |
| 10                   | 9          | 15.00          | 85.00               | $(0.82, 1]$                      | 0.91      |
| 11                   | 4          | 6.67           | 91.67               | $(0.67, 0.82]$                   | 0.75      |
| 12                   | 2          | 3.33           | 95.00               | $(0.54, 0.67]$                   | 0.61      |
| 13                   | 0          | 0.00           | 95.00               | $(0.43, 0.54]$                   | n.a.      |
| 14                   | 3          | 5.00           | 100.00              | $(0, 0.43]$                      | 0.22      |

*Note: 'Cum. Percentage' represents the cumulative percentage. Where the value for  $\lambda$  is 'n. a.', no subject accepts the corresponding number of lotteries.*

#### 1.4.2 Risk aversion

Becker, DeGroot, and Marschak (1964, BDM) originally devised a method to determine a monetary equivalent of a wager. Harrison (1986) subsequently applied this method to elicit a subject's risk aversion attitude. The basic idea of the BDM method is to endow the subject with a series of predetermined lotteries and ask him for a selling price for each lottery with the acknowledgment that a buying price is generated randomly irrespective of the selling price he asks. By this method the subject has an incentive to truthfully reveal the certainty equivalent (CE) of a given lottery.

Before computing each subject's risk aversion coefficient, it is useful to statistically compare the CE and the expected value for the 11 lotteries. We report the corresponding figures as well as the results from Kocher, Pahlke, and Trautmann's (KPT) (2010) experiment in Table 1.4.2. Four out of 11 lotteries' average CEs are greater than the corresponding expected values. With regards to KPT's experiment, all the 11 lotteries' CEs are smaller than the corresponding expected values.

Table 1.4.2 The average certainty equivalent for each lottery in our experiment and KPT's experiment

| Lottery | Expected Value | Average CE  | Average CE (KPT) |
|---------|----------------|-------------|------------------|
| #1      | 8.76           | <b>8.80</b> | 7.82             |
| #2      | 5.30           | 4.99        | 5.00             |
| #3      | 6.70           | <b>6.82</b> | 6.03             |
| #4      | 4.52           | 4.29        | 4.10             |
| #5      | 9.22           | <b>9.59</b> | 8.54             |
| #6      | 5.06           | 4.83        | 4.70             |
| #7      | 4.36           | 4.05        | 3.94             |
| #8      | 8.20           | <b>9.08</b> | 7.83             |
| #9      | 7.76           | 7.53        | 7.22             |
| #10     | 9.76           | 9.66        | 8.93             |
| #11     | 5.01           | 4.76        | 4.68             |

*Note: 'Average CE' is the average certainty equivalent in our experiment; 'Average CE (KPT)' stands for the average certainty equivalent in KPT (2010). Figures in bold font are greater than the corresponding expected values.*

Next, it is necessary to identify the extent of each subject's risk aversion coefficient within the expected utility framework. We denote the utility function when a subject receives money  $w$ :

$$u(w) = w^r, r > 0$$

In such a utility function, the notation  $r$  is the risk preference parameter whereas  $(1 - r)$  is the Arrow-Pratt measure of the relative risk aversion coefficient.<sup>11</sup> If a subject states a selling price  $p_s$  for a lottery with a 50% chance of getting a high payoff  $H$  and a 50% chance of getting a low payoff  $L$ , then it must be that the utility of the monetary payoff  $p_s$  is the same as the utility from the risky lottery, such that:

$$u(p_s) = \text{prob}(H)u(H) + \text{prob}(L)u(L)$$

$$p_s^r = 0.5H^r + 0.5L^r$$

As per KPT (2010, p. 13) we also use a nonlinear least squares technique to estimate each subject's risk preference coefficient  $r$  based on the selling price  $p_s$  that he states, as well as the given lottery's high payoff  $H$  and the low payoff  $L$ . The model we use is as follows:

$$p_{s_i} = (0.5H^{r_i} + 0.5L^{r_i})^{\frac{1}{r_i}} + u_i \quad (1.4.2)$$

in equation (1.4.2), the normal distribution error term  $u_i$  has a property of mean zero and variance  $\sigma^2$

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<sup>11</sup> Relative risk aversion coefficient is calculated as follows:  $R(w) = -w \cdot \frac{u''(w)}{u'(w)}$ .

Table 1.4.3 Risk preference classification and the corresponding number of subjects

| Range of risk preference | # Subjects (Percentage) | Total       | KPT: # Subjects (Percentage) | Total       | Risk preference classification |
|--------------------------|-------------------------|-------------|------------------------------|-------------|--------------------------------|
| (1.95, $\infty$ )        | 15 (25%)                | 27          | 3 (2%)                       | 27          | highly risk loving             |
| (1.49, 1.95]             | 5 (8.33%)               | (45%)       | 6 (4%)                       | (18%)       | very risk loving               |
| (1.15, 1.49]             | 7 (11.67%)              |             | 18 (12%)                     |             | risk loving                    |
|                          |                         | 11 (18.33%) |                              | 34 (22.67%) |                                |
| (0.85, 1.15]             | 11 (18.33%)             |             | 34 (22.67%)                  |             | risk neutral                   |
| (0.59, 0.85]             | 6 (10%)                 |             | 23 (15.33%)                  |             | slightly risk averse           |
| (0.32, 0.59]             | 3 (5%)                  | 22          | 17 (11.33%)                  | 89          | risk averse                    |
| (0.03, 0.32]             | 2 (3.33%)               | (36.66%)    | 18 (12%)                     | (59.33%)    | very risk averse               |
|                          |                         |             |                              |             | highly risk averse             |
| (-0.37, 0.03]            | 3 (3.33%)               |             | 15 (10%)                     |             |                                |
| ( $-\infty$ , 0.37)      | 9 (15%)                 |             | 16 (10.67%)                  |             | stay in bed                    |

*Note: We obtained KPT's experiment data from Appendix D. of ScienceDirect website <http://www.sciencedirect.com/science/article/pii/S0014292115000677>.*

In Table 1.4.3, we report the estimated range of the risk preference coefficient  $r$  and the corresponding number of subjects in our experiment as well as in KPT's experiment.<sup>12</sup> Here, we follow the risk preference classification as per Holt and Laury (2002). In our experiment of 60 subjects, 45% of them are risk loving; whereas 11 subjects (18.33%) are risk neutral, and the remaining 22 subjects (36.66%) are risk averse.

<sup>12</sup> KPT does not report subjects' risk aversion preferences in both the 2010 and 2015 papers. However, we can obtain such results using the data and the code they provide in the 2015 paper.

The results from KPT's experiment are inconsistent with our finding. That is, the majority of subjects are risk averse (59.33%) and only 18% of subjects are risk loving while 22.67% of subjects are risk neutral.<sup>13,14</sup>

After identifying each subject's loss aversion coefficient  $\lambda$  and risk aversion coefficient  $(1 - r)$ , we wonder whether these two coefficients are related to each other as Thaler et al. (1997) suggest. In their experiment, the subjects need to make some investment decisions between two funds – bond and stock funds, within four conditions – monthly, yearly, five-yearly, and inflated monthly. A major conclusion they get is: “Investors who display myopic loss aversion will be more willing to accept risks if they evaluate their investments less often.” In this BDM lottery experiment, if we consider a lottery decision as an investment, the subject can only know the result of the investment at the very end of the experiment. This prohibits them from adopting a ‘narrow framing’ as defined by Kahneman and Lovallo (1993) – in other words, considering decision problems one at a time. As a result, it is not very surprising that 45% of subjects are risk loving.

In order to examine whether the two estimation parameters are correlated, we create a scatter plot of the  $(1 - r)$  and  $\lambda$  along with histograms of the two variables as in Figure 1.4.2.<sup>15</sup> We can see that there is no clear linear correlation between these two variables.<sup>16</sup>

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<sup>13</sup> We have excluded the possibility that the difference is due to the 11 lotteries being presented to the subjects in a different manner between our experiment and KPT's. For both experiments, the 11 lotteries are shown to the subjects on 11 separate pages.

<sup>14</sup> However, these results are close to the findings reported by Berg, Dickhaut and McCabe (2003). In their design, the basic essence of the BDM method is the same. But instead of a 50-50 lottery, they use a 30-sided die and a cut-off value  $p$  to decide the payoff for the subject if his selling price is above the randomly generated buying price. Hence, they find that within 48 subjects, about 55% of them are risk loving.

<sup>15</sup> In figure 1.4.2, we have eliminated two outliers with an extremely large negative risk aversion coefficient (-273.6), which shows that the corresponding subjects are extremely risk loving. In the BDM lottery stage, the two subjects both stated a selling price  $H$  for all the 11 lotteries.

<sup>16</sup> We also cannot observe a linear relationship when we use  $(\sum p_s, \text{accepted lotteries})$  as variables to plot the chart.

In terms of the risk aversion coefficient, most subjects cluster in the range of  $[-2, 2]$ . With regards to the loss aversion coefficient, the majority of subjects are between 0 and 4.<sup>17</sup>

Our result is very different from the result reported by Goldstein, Johnson, and Sharpe (2008), in which they find that for the 570 subjects in their experiment, the estimates of the risk aversion and loss aversion parameters are correlated.<sup>18</sup> Most of their subjects displayed a low risk aversion, as well as a low loss aversion attitude. It is acknowledged that besides the distinction of the two experimental designs, the inconsistency of the results could be due to sample size differences.

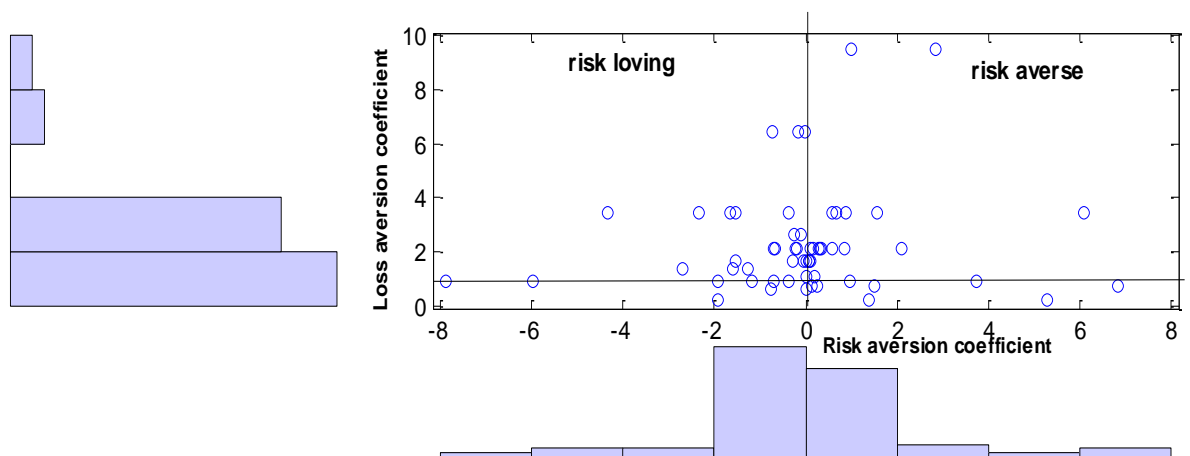


Figure 1.4.2 Graphical illustration of the relationship between loss aversion and risk aversion coefficients using a scatter plot and histograms.

## 1.5 Experimental results for auction stages

### 1.5.1 Modelling bid behaviour

<sup>17</sup> As a robustness check, the nonparametric Spearman test shows that for the 58  $(1-r, \lambda)$  pairs, the two variables are independent from each other ( $p$ -value  $> 0.1$ ).

<sup>18</sup> The two variables have a very clear linear correlation not only from the chart but also verified by a Pearson correlation test.

In order to identify how the payback scheme works in the first-price auctions, in this section we use a panel data regression approach to estimate the aggregate bid functions.

As mentioned before, we use a within-subject design for the auction experiment, in which each subject experiences two first-price auction treatments: standard (k0) and a novel payback scheme (k5). Neugebauer and Perote (2008) also use a within-subject design to compare the bids of first-price auctions in two treatments: with and without the information feedback. In this chapter we follow their method to model the bidding behaviour. The model is as follows

$$bid_{kit} = \beta_0 + \beta_1 Dk_{kit} + \beta_2 pv_{kit} + \beta_3 Dk_{kit} pv_{kit} + v_k + \varepsilon_{kit} \quad (1.5.1)$$

In equation (1.5.1),  $bid_{kit}$  and  $pv_{kit}$  denote the bid and the private value of subject  $i$  of group  $k$  in round  $t$ , where  $i = \{1, 2, \dots, 6\}$ ,  $k = \begin{cases} 1, 2, 3 & n = 6 \\ 1, 2 & n = 3 \end{cases}$ ,  $t = \{1, 2, \dots, 40\}$ .  $\beta_j$  are the parameters to be estimated,  $j = \{0, 1, 2, 3\}$ ,  $\varepsilon_{kit}$  is an error term, which is assumed to have mean zero and variance  $\sigma_\varepsilon^2$ ;  $v_k$  is the group-specific term. This model accounts for the possible structural changes between the k0 and k5 treatments by using a dummy variable  $Dk_{kit}$ , which takes the value one for the k5 treatment and zero for the k0 treatment. Since this dummy variable interacts with both the intercept and the slope, we can interpret the results from Table 1.5.1 as the bid functions for each treatment.

As in Section 1.2, we derive the RNNE and LASNE bidding strategies in the case of first-price private value auctions with a payback scheme as follows

$$b(v_i)^{RNNE} = K + \frac{n-1}{n} v_i$$

$$b(v_i)^{LASNE} = \lambda K + \frac{n-1}{n} v_i$$



As illustrated in Section 1.4.1, the subjects are loss averse on average ( $\lambda > 1$ ). Therefore, if the subjects bid according to the RNNE or LASNE model, we have the following hypotheses

$$H1_0: \beta_1 \geq 5$$

$$H2_0: \beta_3 = 0$$

The null hypothesis is that the payback scheme should only influence the intercepts while not affecting the slopes. We start by discussing the estimated intercepts. From the coefficients of the dummy variable  $Dk_{kit}$  shown in Table 1.5.1 below, we can find that  $\beta_1 < 5$  which implies that the subjects would not use all of the \$5 given to them to submit their bids in the payback auctions. In three of the four sessions, on average, subjects use around \$4 (80% of \$5) to bid. For the  $k5_1$  treatment in the 3-bidder market, the subjects on average only use around \$3 (60% of \$5) to bid. This result suggests that the RNNE and LASNE models both fail to explain the realized bids for the payback first-price auctions, as they overestimate the impact of the payback scheme on the intercept.

Table 1.5.1 also reveals that  $\beta_3$  is significantly positive in the last session:  $k5k0\_3$ . Bids in the payback scheme treatment involve a significantly higher fraction (0.153) of private value than the standard first-price treatment, which is inconsistent with the RNNE and LASNE predictions.

**Result (payback scheme effect):** in all four sessions of the payback first-price auctions, the subjects use some but not all of the initial capital balance  $k$  to submit bids. Furthermore, the subjects reveal a higher fraction of their private values in the session with a 3-bidder market where subjects are exposed to the payback scheme before experiencing the standard auction.

Table 1.5.1 Coefficients of random effect regression: linear bid function

| Independent Variable   | Dependent variable: bid |                   |                   |                   |
|------------------------|-------------------------|-------------------|-------------------|-------------------|
|                        | n=6                     |                   | n=3               |                   |
|                        | k0k5                    | k5k0              | k0k5              | k5k0              |
| Intercept              | -0.391<br>(0.277)       | -0.313<br>(0.185) | -0.079<br>(0.209) | 0.002<br>(0.001)  |
| Dk                     | 4.065*<br>(0.306)       | 4.067*<br>(0.535) | 4.097*<br>(0.304) | 2.914*<br>(0.172) |
| pv                     | 0.878*<br>(0.030)       | 0.948*<br>(0.014) | 0.702*<br>(0.048) | 0.790*<br>(0.032) |
| Dk X pv                | 0.050<br>(0.099)        | 0.016<br>(0.062)  | 0.093<br>(0.059)  | 0.153*<br>(0.074) |
| R <sup>2</sup> overall | 0.762                   | 0.840             | 0.887             | 0.736             |
| # Observations         | 720                     | 720               | 480               | 480               |
| # Groups               | 3                       | 3                 | 2                 | 2                 |

*Note: estimate for equation 1.5.1, (robust standard error in parentheses); \* significant at 5%.*

After identifying the treatment effect of the novel payback scheme, it is also instructive to analyse the bidding behaviour in the control treatment: standard first-price auctions. If the subjects bid according to the RNNE model, we have the following hypotheses

$$H3_0: \beta_0 = 0$$

$$H4_0: \beta_2 = 0.83 \ (n = 6)$$

$$\beta_2 = 0.67 \ (n = 3)$$

From Table 1.5.1, we can verify that  $H3_0$  is correct. At the same time, subjects' bids as a fraction of private value are substantially greater in the  $k0_2$  treatment compared with the  $k0_1$  treatment for both the 3-bidder and 6-bidder markets. It may be that subjects get used to submitting high bids during the payback scheme and as a result continue to submit high bids once the payback scheme is removed ('anchoring').<sup>19</sup> Therefore, we observe an order effect in bidding behaviour for the  $k0$  treatment.

**Result (order effect in the  $k0$  treatment):** for the standard first-price auctions, the subjects bid a higher fraction of their private values if they experience the payback scheme first.

When we take a closer look at  $\beta_2$  in four sessions, we cannot reject  $H4_0$  for the two  $k0_1$  sessions. However, we have to reject  $H4_0$  in favour of the alternative hypothesis that  $\beta_2$  is greater than the corresponding RNNE prediction for the two  $k0_2$  sessions. Overall, in the standard first-price auctions where the subjects have no experience of the payback scheme, the bidding behaviour can be explained by the RNNE prediction; for the subjects who have experienced the payback scheme in advance, their bids exceed the RNNE prediction.

**Result (bid vs RNNE prediction in the  $k0$  treatments):**

$$\text{For the } k0_1 \text{ treatment: } bid_{n=6} = bid_{n=6}^{RNNE}; bid_{n=3} = bid_{n=3}^{RNNE}$$

$$\text{For the } k0_2 \text{ treatment: } bid_{n=6} > bid_{n=6}^{RNNE}; bid_{n=3} > bid_{n=3}^{RNNE}$$

---

<sup>19</sup> We do not find any evidence that learning (experiment rounds) plays a role in bidding behaviour, which may be because in our experiment, each treatment only lasts for 20 rounds.

We also use Figure 1.5.1 to demonstrate the relationships between the realized bids and the corresponding RNNE predictions for two markets in standard auctions while considering the order effect. Such plots clearly show an overbidding pattern for the  $k_0$  treatment.

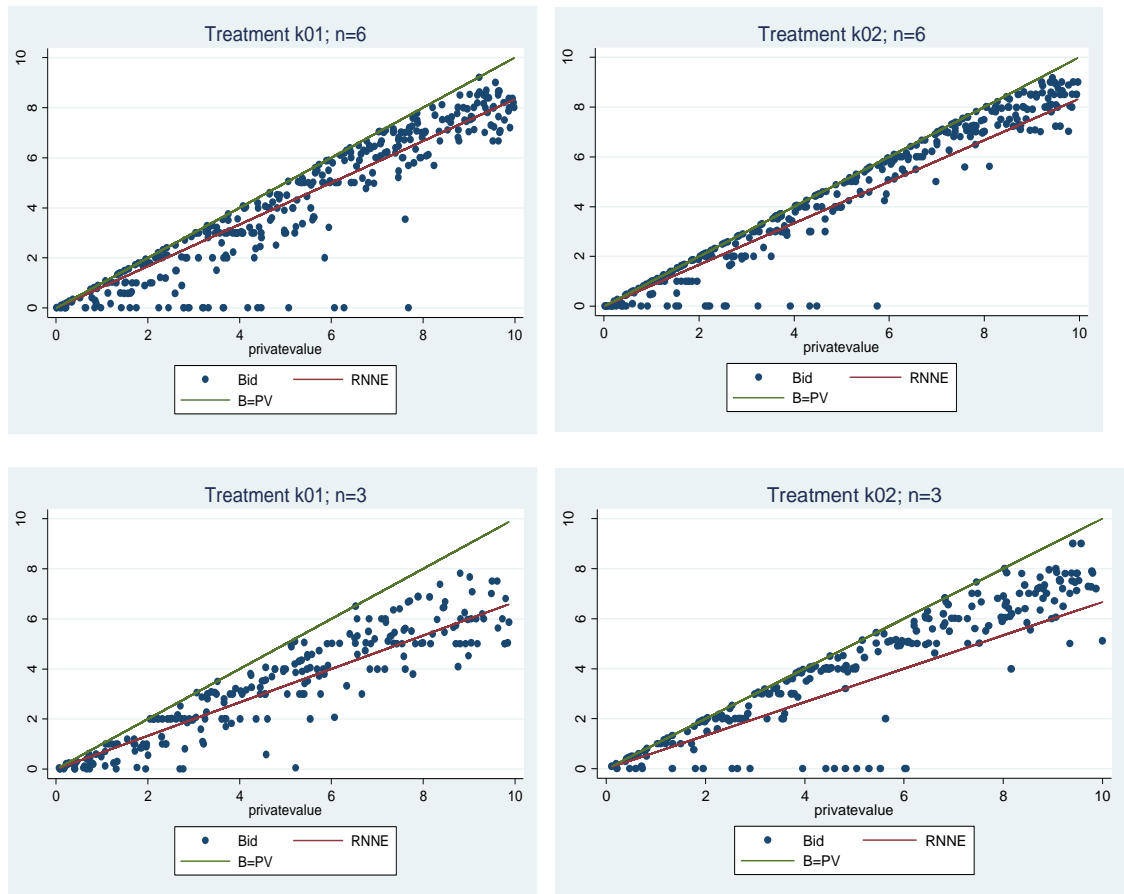


Figure 1.5.1 The bids and the RNNE predictions in the  $k_0$  treatment

### 1.5.2 Seller's revenue and the allocation efficiency

Revenue and efficiency are the two main measurements for evaluating an auction format. Since in this chapter, the major research question is whether or not applying a payback

scheme can enhance the seller's revenue, we first analyse the realized revenue by checking the two hypotheses in Section 1.2.4.

Before using econometric methods to verify the conjectures, we first report the revenue statistics in Table 1.5.2.<sup>20</sup> From Table 1.5.2, we can observe that the k0<sub>2</sub> treatment brings the greatest revenue for both market sizes on average (7.93 and 6.32, respectively). Besides this, between the two market sizes, the revenue in the 6-bidder market is always greater. Such findings are consistent with our results for the estimated bidding functions as illustrated in the previous section.

Table 1.5.2 The statistics of average revenue and efficiency of the two treatments for both market sizes

|     | Treatment | Batch | # Observations | Revenue |      | Efficiency |      |
|-----|-----------|-------|----------------|---------|------|------------|------|
|     |           |       |                | Mean    | S.D. | Mean       | S.D. |
| n=6 | k0        | 1     | 60             | 7.39    | 1.02 | 98.21%     | 0.05 |
|     |           | 2     | 60             | 7.93    | 0.96 | 98.56%     | 0.04 |
|     | k5        | 1     | 60             | 7.5     | 1.43 | 97.48%     | 0.06 |
|     |           | 2     | 60             | 7.06    | 1.14 | 93.59%     | 0.16 |
| n=3 | k0        | 1     | 40             | 5.11    | 0.97 | 96.92%     | 0.08 |
|     |           | 2     | 40             | 6.32    | 1.10 | 95.14%     | 0.14 |
|     | k5        | 1     | 40             | 5.4     | 1.64 | 88.91%     | 0.19 |
|     |           | 2     | 40             | 5.12    | 1.20 | 93.42%     | 0.12 |

*Note: With respect to the third column 'Batch': '1' and '2' represent the corresponding treatment in auction rounds 1-20 and 21-40, respectively. 'Mean' is obtained by taking the average of each group across 20 rounds with the specific treatment and batch. 'S.D.' is the standard deviation of the average.*

In this section, we use a random effect panel data regression model similar to that used by Schram and Onderstal (2009), which includes variables for both treatment effect and order effect. In this chapter, the model explaining realized revenue is given by:

<sup>20</sup> With the payback scheme, since we need to take the \$5 given to the winner into consideration, the realized revenue=winner's bid-\$5.

$$R_{kt} = \beta_0 + \beta_1 Dk_{kt} + \beta_2 Order_{kt} + u_k + \varepsilon_{kt} \quad (1.5.2)$$

where dummy variable  $Order_{kt} = \begin{cases} 0, & k0k5 \\ 1, & k5k0 \end{cases}$ . The other variables are the same as in equation (1.5.1). Therefore, the control treatment is the standard first-price auction with no experience about the payback scheme (k0<sub>1</sub>). Table 1.5.3 shows the results. The estimated coefficient of  $Dk_{kt}$  is significantly negative in the 6-bidder market (-0.38) whereas it is insignificant in the 3-bidder market.

***Revenue Result (payback scheme effect): In the 6-bidder market, the seller's revenue is smaller with the payback scheme whereas it is not significantly different in the 3-bidder market.***

$$R_{k5}^{n=6} < R_{k0}^{n=6}; R_{k5}^{n=3} = R_{k0}^{n=3}$$

At the same time, we can see that the coefficient of  $Order_{kt}$  is significantly positive for both market sizes, indicating that the revenue for the standard first-price auction is greater when the subjects have experienced the payback scheme. This result is consistent with our finding in the previous section that the estimated slope is greater in the k0<sub>2</sub> treatment compared to the k0<sub>1</sub> treatment. Furthermore, this coefficient is larger in the 3-bidder market. Figure 1.5.2 plots the difference between the realized price and the RNNE predicted price for each round in both the 3- and 6-bidder markets in the corresponding k0<sub>1</sub> and k0<sub>2</sub> treatments. We can see that for the k0<sub>2</sub> treatment, the differences between the realized prices and the RNNE predicted prices are invariably above zero, especially for the 3-bidder market, and such differences are much greater than in the k0<sub>1</sub> treatment.

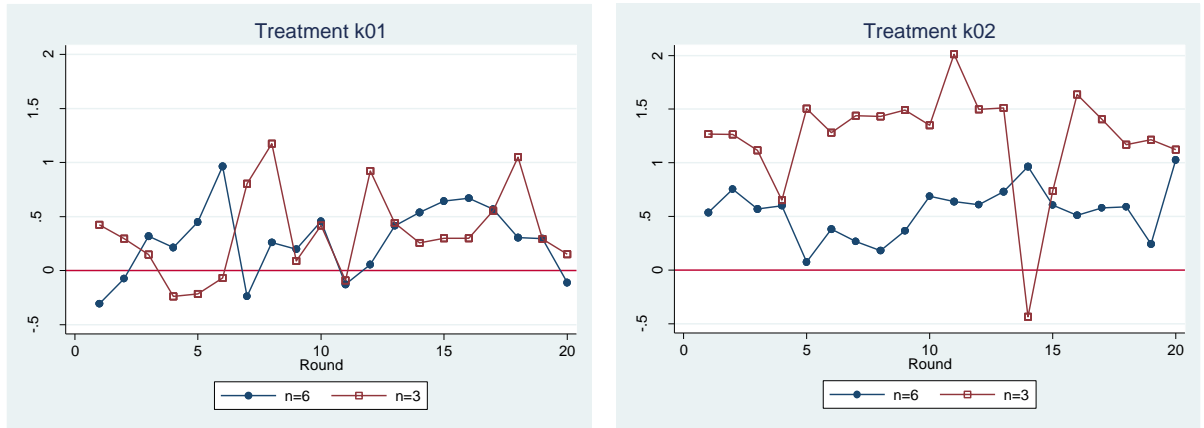


Figure 1.5.2 The difference between observed prices and RNNE prices in the k0<sub>1</sub> and k0<sub>2</sub> treatments

*Note: In the 6-bidder market, for each auction round in a given treatment we compute the average difference between the realized prices and the RNNE-predicted prices across the associated 3 groups. In the 3-bidder market, we follow the same method, except that there are only 2 groups in a given treatment.*

**Revenue Result (order effect):** *In the standard auction format, the seller's revenue is increased if bidders have experienced the payback scheme in advance; moreover, this revenue increasing is greater in the 3-bidder market.*

$$R_{k0_2} > R_{k0_1}$$

$$(R_{k0_2} - R_{k0_1})_{n=3} > (R_{k0_2} - R_{k0_1})_{n=6}$$

There are two earlier papers which also compare the realized prices with the RNNE prediction in first-price auctions of 3-bidder and 6-bidder markets. Cox, Roberson, and Smith (1982) find that the RNNE prediction cannot be rejected for the 3-bidder market whereas in the 6-bidder market, overbidding behaviour is observed. However, Dyer, Kagel, and Levin (DKL) (1989) identify a different result, which is that the winning bids exceed the RNNE prediction for both 3- and 6-bidder markets.<sup>21</sup>

<sup>21</sup> DKL (1989) use a within-subject design that each subject submits two contingent bids for 3- and 6-bidder markets.

Besides revenue differences due to the treatment effect and the order effect, **Hypothesis 2**:  $R_{n=6} > R_{n=3}$  is easily verified as the coefficient of the intercept term is significantly greater in the 6-bidder market.

**Revenue Result (6-bidder market vs 3-bidder market):**  $R_{n=6} > R_{n=3}$

Table 1.5.3 Coefficients of random effect and pooled Ordinary Least Squares (OLS) regressions for two market sizes

| Independent Variable | Dependent variable: Revenue |                   |                  |
|----------------------|-----------------------------|-------------------|------------------|
|                      | n=6                         |                   | n=3              |
|                      | RE model                    | Pooled OLS        | RE model         |
| Intercept            | 7.41*<br>(0.138)            | 7.41*<br>(0.136)  | 5.35*<br>(0.159) |
| Dk                   | -0.37*<br>(0.075)           | -0.38*<br>(0.078) | -0.50<br>(0.330) |
| Order                | 0.49*<br>(0.153)            | 0.49*<br>(0.150)  | 0.77*<br>(0.104) |
| #Observations        | 240                         | 240               | 160              |
| BP test              | p=0.32                      |                   | p<0.05           |

*Note: estimate for equation (1.5.2), (robust standard error in parentheses); \* significant at 5%. For each revenue observation with 3-bidder market of each group  $k$ , in each round  $t$ , we compute the average revenue for two subgroups. The BP test (Breusch and Pagan test) for random effect tests  $\text{var}(u_k) = 0$ , is rejected for the 3-bidder market, but not for the 6-bidder market.<sup>22</sup>*

<sup>22</sup> For the panel data analysis, when choosing between different panel models, we need to test for the individual specific effects. The null hypothesis for the BP test is the individual specific variance component is zero. In the 6-bidder market p-value=0.32, we cannot reject the null hypothesis, which suggests that there is no significant random effect and the Pooled OLS model is preferred than the random effect model.



As a result, the payback scheme fails to increase the seller's revenue, which differs from the theoretical model prediction. However, we find that the seller's revenue can be increased in the standard first-price auction if the subjects have experienced the payback scheme before. Therefore, even if the payback scheme itself does not enhance the revenue, including this scheme as a trial session before the standard first-price auctions, will give the subjects the inertia to submit a higher bid.

Having addressed the revenue comparison between treatments, next we look at the second measurement - allocation efficiency for the two treatments. To determine the allocation efficiency, we compute the following equation which represents the percentage of surplus captured by an auction mechanism

$$e = \frac{pv_{winner}}{pv_{highest}} \quad (1.5.3)$$

where  $pv_{winner}$  stands for the winner's private value and  $pv_{highest}$  is the highest private value. We report the corresponding results in the last column of Table 1.5.2. On average, the auction is more efficient in the k0 treatment for both market sizes. This is not surprising, because in the payback scheme auction, the bidders can bid up to their private values plus \$5. It gives the bidder an opportunity to win the auction who uses a larger proportion of the \$5 when submitting his bid, even though his private value is not the highest.

## 1.6 Risk preferences in different institutions – first-price auctions and BDM lottery

A number of papers (such as Isaac & James, 2000; Neugebauer & Selten, 2006) identified an overbidding phenomenon and use risk aversion to explain it. In addition, they find out what is the individual's risk preference parameter  $r$  in first-price auctions.

In this section, we will qualitatively compare the risk parameters for each group within our two experimental institutions: the first-price auction and the BDM lottery. Here, 'first-price auction' refers to the k0 treatment only. As we have shown in Section 1.5.1, for the k5 treatment, the estimated intercept of the bid function is significantly less than k, which goes against the RASNE and the LASNE predictions. Following Isaac and James (2000), Engel (2009), and Neugebauer and Selten (2006), we back out the risk parameter  $r_i$  using the observations of bids and private values for the specific market size within the RASNE model. However, unlike these papers, we do so for each group instead of each subject. This is because in this chapter our homogeneous risk preference assumption is different from their heterogeneity assumption.<sup>23</sup>

We have derived the RASNE bidding strategy in the standard first-price auction as follows

$$b(v_i) = \frac{n-1}{n+r-1} v_i \quad (1.6.1)$$

Therefore, the bid data is used to estimate the linear bid function below, and we remove the 'zero' bids from the observations.

$$b_i = \alpha_i + \beta_i p v_i + e_i \quad (1.6.2)$$

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<sup>23</sup> However, we acknowledge that our experiment design does not guarantee this assumption holds since the subjects formed in one group are randomly chosen.

Where under the RASNE model, the prediction is that  $\alpha_i = 0, \beta_i = \frac{n-1}{n+r-1}$ , from which

we can obtain the group risk preference parameter  $r$  as follows:

$$r = \frac{(1 - \beta_i)(n - 1)}{\beta_i} \quad (1.6.3)$$

In our data presentation, we restrict our attention to those bidder groups which satisfy the equilibrium condition that  $\alpha_i$  is not significantly different from zero. We report the results of the estimated group risk preference parameters using equation (1.6.3) in Table 1.6.1. As can be seen from the table, the estimated intercepts in groups 7 and 9 are significantly positive. Therefore, we do not consider the risk parameters in these two groups. Within the eight groups for which the risk preference parameters  $r$  can be estimated, most groups display different levels of risk aversion in the auction task.<sup>24</sup> In Table 1.6.1, we also report the corresponding results from the BDM lottery task. As we have shown in Section 1.4.2, most groups are risk loving in the BDM lottery stage. Overall, our study confirms the instability of risk parameters across different institutions as widely observed by a number of papers, e.g. Isaac and James (2000), Anderson and Mellor (2009), and Hey, Morone, and Schmidt (2009).<sup>25</sup>

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<sup>24</sup> We use the same classification as in Section 1.4.2.

<sup>25</sup> Different from Isaac and James (2000), which also reported the instability of risk preferences between the first-price auction task and the BDM selling procedure, Anderson and Mellor (2009) identify the instability between monetary rewards experiment and a job-based gambles survey; Hey, Morone and Schmidt (2009) report such instability across four incentive-compatible elicitation methods.

Table 1.6.1 Risk preference parameters in the auction and BDM tasks for each group

| Group | Auction task (k=0) |               |           |                      |        | BDM task  |                      |
|-------|--------------------|---------------|-----------|----------------------|--------|-----------|----------------------|
|       | $\hat{\alpha}$     | $\hat{\beta}$ | $\hat{r}$ | Classification       | # Obs. | $r\_mean$ | Classification       |
| 1     | 0.126              | 0.861*        | 0.807     | slightly risk averse | 114    | 1.043     | risk neutral         |
| 2     | -0.325             | 0.882*        | 0.669     | slightly risk averse | 102    | 0.701     | slightly risk averse |
| 3     | -0.115             | 0.835*        | 0.988     | risk neutral         | 113    | 1.340     | risk loving          |
| 4     | -0.180             | 0.888*        | 0.631     | slightly risk averse | 107    | 1.126     | risk neutral         |
| 5     | 0.102              | 0.907*        | 0.513     | risk averse          | 114    | 1.221     | risk loving          |
| 6     | -0.051             | 0.951*        | 0.258     | very risk averse     | 111    | 1.407     | risk loving          |
| 7     | 0.210*             | 0.646*        | n.a.      | n.a.                 | 117    | 2.519     | highly risk loving   |
| 8     | -0.321             | 0.756*        | 0.646     | slightly risk averse | 113    | 1.850     | very risk loving     |
| 9     | 0.388*             | 0.748*        | n.a.      | n.a.                 | 106    | 1.614     | very risk loving     |
| 10    | 0.211              | 0.800*        | 0.500     | risk averse          | 115    | 3.990     | highly risk loving   |

*Note: \*significant at 5%. Each group includes 6 subjects who submit bids for 20 rounds in standard auctions, so there are  $6 \times 20 = 120$  observations for each group. After censoring the 'zero' bids, as shown in the column '#Obs.', the applicable number of observations is less than 120 for each group in the standard auction task.*

## 1.7 Relevant research

Delgado, Schotter, Ozbay, and Phelps (2008) report a similar experimental design which combines neuroeconomic and behavioural economic techniques. Behavioural economic techniques analyse subjects' experimental decisions to test theoretical predictions. The neuroeconomic approach extends this field by adding observations from the nervous system. Using the finding of neural circuitry, they conjecture that by 'manipulating the parameters of a first-price auction to highlight the potential for loss, it would not only increase bids, but also raise more revenue.' Therefore, they conduct three treatments in a first-price auction format: baseline, loss-frame, and bonus-frame.

We report the differences of the experimental designs between this chapter and their article in Table 1.7.1. Delgado et al. (2008) adopt a between-subject design, which means each subject only participates in one treatment. Another major difference is that in their experiment, the amount of initial capital balance  $K$  given to each subject before the auction starts is only 15% relative to the maximum private value, compared with 50% in our experiment.

Table 1.7.1 First-price auction experimental designs in this chapter and in Delgado et al. (2008).

| Treatment               | Our Experiment                    |                | Delgado et al. (2008)         |            |             |
|-------------------------|-----------------------------------|----------------|-------------------------------|------------|-------------|
|                         | Standard                          | Payback scheme | Baseline                      | Loss-frame | Bonus-frame |
| K                       | 0                                 | 5              | 0                             | 15         | 15          |
| Randomisation technique | Within-subject                    |                | Between-subject               |            |             |
| N (Matching method)     | 6 (Partner)<br>3 (Partner+Random) |                | 2 (Random)                    |            |             |
| Rounds                  | 20                                |                | 30                            |            |             |
| PV                      | AUD[0,10]                         |                | [0, 100] Experimental dollars |            |             |

*Note: In Delgado et al. (2008), \$1 USD = 60 experimental dollars.*

The main finding of their experiment is that the seller's revenue is higher in the 'loss-frame' treatment compared with the baseline treatment. Such results are very intriguing since in our experiment, the seller's revenue is significantly smaller in the payback scheme auction relative to the standard first-price auction for the 6-bidder market, and not significantly different for the 3-bidder market, as we illustrated in section 1.5.2. In order to explore the reason why the schemes affect revenue differently, we compare Delgado et al.'s (2008) experiment results with our findings by re-estimating the bidding functions through normalized bids and private values in the unit interval, and also removing all the 'zero' bids. Here, by pooling all the observations in the same market size and treatment together while ignoring the order effect, we report the corresponding results in Table 1.7.2.

We need to mention that we could only obtain the bid data of 34 out of the 52 subjects who participated in the 'loss-frame' treatment in Delgado et al. (2008). Using the

available data, we obtain similar results to those reported by Delgado et al. (2008) – after normalizing, the average revenue is 0.456 and the random effect bid function is  $b = 0.111 + 0.74pv$ . However, analysis of the baseline treatment in Delgado et al. (2008), due to the missing data issue could not be undertaken. Therefore, we choose to report the baseline treatment regression results as Delgado et al. (2008) provided in their paper.

Table 1.7.2 The mean revenue and estimated random effect bid functions from our experiment and Delgado et al. (2008)’s experiment

| n |               | standard FP auction | payback scheme FP auction |
|---|---------------|---------------------|---------------------------|
|   | Revenue       | 0.409               | 0.454                     |
| 2 | RNNE bid      | $b^*=0.500pv$       | $b^*=0.150+0.500pv$       |
|   | Estimated Bid | $b=0.614pv$         | $b=0.109+0.733pv$         |
|   | # Obs.        | 660                 | 1018                      |
|   | $R^2$         | 0.805               | 0.757                     |
|   | Revenue       | 0.571               | 0.526                     |
| 3 | RNNE bid      | $b^*= 0.67pv$       | $b^*=0.500+0.67pv$        |
|   | Estimated Bid | $b=0.740pv$         | $b=0.387+0.810pv$         |
|   | # Obs.        | 451                 | 480                       |
|   | $R^2$         | 0.862               | 0.631                     |
|   | Revenue       | 0.766               | 0.728                     |
| 6 | RNNE bid      | $b^*= 0.83pv$       | $b^*=0.500+0.83pv$        |
|   | Estimated Bid | $b=0.886pv$         | $b=0.430+0.896pv$         |
|   | # Obs.        | 661                 | 681                       |
|   | $R^2$         | 0.945               | 0.745                     |

*Note: ‘FP auction’ indicates first-price auction. In Delgado et al. (2008), ‘standard FP auction’ refers to the baseline treatment whereas ‘payback scheme FP auction’ refers to the loss-frame treatment. The figure of ‘# Obs.’ in each market size for the corresponding treatment is obtained by deleting all the ‘zero’ bids from the total number of bids.*

Table 1.7.2 clearly shows a common pattern for the estimated bid functions in our experiment and Delgado et al. (2008)’s experiment. That is, in the payback scheme first-

price auctions, the estimated bid intercepts are always significantly below the RNNE predictions. With regards to the bid slopes, they are all greater in the payback scheme first-price auctions than the standard first-price auctions, but are only significantly greater in Delgado et al. (2008)'s experiment with a 2-bidder market. Therefore, by comparing the corresponding estimated bid intercepts and slopes, we obtain two possible explanations of the different revenue results within the payback scheme first-price auctions. Considering that the estimated intercept would be always smaller than  $K$ , a necessary requirement of the payback scheme enhancing the seller's revenue is that the slope coefficient must be substantially greater compared with that in the standard first-price auction. This is quite unlikely in the larger market sizes, because as market size increases, the slope coefficient already gets closer and closer to 1, and hence does not have much room to keep increasing.

Compared with market size, determining a proper amount of initial capital balance  $K$  is likely to play a more important role in whether the scheme enhances the seller's revenue. We can see from both the experiment results that, such a scheme no doubt increases bids regardless of the amount of  $K$ . However, in our experiment,  $K$  is set too high (50%) relative to the maximum possible private value  $\bar{v}$ . Hence, even though bids increase due to the payback scheme, they increase by less than  $K$  and hence revenue decreases.

As a result, by combining the results of Delgado et al. (2008) experiment with our findings, we obtain the following two conditions, under which the seller's revenue may increase in a payback scheme first-price auction:

- Small market size
- Initial capital balance  $K < 0.5 \bar{v}$ .



## 1.8 Concluding remarks

The purpose of this chapter is to examine whether a payback scheme in first-price private value auctions could enhance seller's revenue due to the existence of loss aversion. We derive a simple single-unit first-price private value auction model, which encompasses two cases - bidders display a homogeneous risk averse attitude or loss averse attitude. Based on the model, the payback scheme should increase the revenue if subjects are loss averse whereas it should not make a difference when subjects are risk averse.

We design an auction experiment using within-subject variation. Each subject participates in two treatments: payback scheme (k5) and standard first-price auction (k0). We also take the order effect into consideration and conduct the experiment with k0k5 and k5k0 orders in two market sizes (6-bidder and 3-bidder). However, the experimental results do not support the hypothesis. More specifically, the revenue within the payback scheme is statistically less than in the standard auction in the 6-bidder market and is not significantly different in the 3-bidder market. Nonetheless, the revenue in the standard auction is increased when subjects have experienced the payback scheme first.

We suggest that the reason the payback scheme fails to enhance revenue in our experiment is that the subjects simply use a certain proportion of the initial capital balance  $K$  when submitting bids regardless of private value, and are not induced to respond more strongly to a marginal increase in private value. This is reflected in the intercept of the bidding function increasing by less than  $K$  and the slope not changing significantly. Therefore, although the subjects submit higher bids, this does not offset the cost of the initial capital balance  $K$  retained by the winner. Combined with the results reported by Delgado et al. (2008) in which the revenue is increased in the 'loss-frame' treatment, a

natural extension to our experiments in the future is to set a smaller  $K$  (e.g. \$1.5) and to test if the payback scheme works or not.

With regards to the experimental design, the future study could also implement another treatment in which only the winner obtains the money  $K$ , whereas all the losers receive nothing. Such an auction design is strategically equivalent to the payback scheme, and it would be interesting to compare the results to this chapter. There is also some scope for future research to extend the theoretical framework to incorporate reference-dependent preferences.

## **Chapter 2: Heterogeneous risk attitudes in first-price independent private value auctions and treatment effect**

### **2.1 Introduction**

Vickrey (1961) was the first to apply game theory to the study of auctions and derived the risk neutral Nash Equilibrium (RNNE) bidding strategy in first-price auctions. This remarkable theoretical paper also suggests the existence of a particular pricing rule that produces the same result as in the traditional English (ascending-bid) auction. That is, the winner pays the second highest bid (second-price auction) and Vickrey also demonstrated the dominant strategy – reporting values truthfully. Within the single-unit independent private value context, the Revenue Equivalence Theorem states that first-price auctions and second-price auctions yield the same expected revenue for the seller in the equilibrium. Actually, this theorem vitally depends on the assumption that all bidders are risk neutral. Vickrey also shows that the first-price auction and Dutch (descending-bid) auction formats are strategically equivalent; that is the bidder should bid strictly less than his willingness to pay, and the amount of shading from his willingness to pay should depend on his belief about his rivals' bids.

The RNNE model was first challenged by Coppinger, Smith, and Titus (1980), who implement a series of experiments and find that not only the winning prices, but also the bids in first-price auctions significantly exceed the RNNE predictions, and that the first-price auction and Dutch auction are not 'isomorphic'. Since then, numerous laboratory experiments have shown that bidders bid significantly higher than the risk neutral prediction (while still below value) in first-price auctions (CRS, 1982; CSW, 1988; Kagel & Roth, 1995; Kagel & Levin, 2011).

There is a huge amount of literature on what drives overbidding behaviour in first-price auctions. In particular, this includes risk aversion (CSW, 1988), Quantal Response Equilibrium (QRE) with risk aversion (Goeree, Holt, & Palfrey, 2002), misperception of probability of winning (Dorsey & Razzolini, 2003; Armantier & Treich, 2009), learning direction theory (Neugebauer & Selten, 2006); anticipation of regret (Filiz & Ozbay, 2007); “missed-opportunity-to-win” regret (Engelbrecht-Wiggans & Katok, 2008); and loss aversion (Lange & Ratan, 2010). Among all of these, risk aversion is the most widely accepted explanation.

Since risk aversion implies a steeper bidding function compared to the risk neutral bidding line, it does give a better approximation of actual bids. The homogeneous risk aversion assumption<sup>26</sup> which requires all the bidders who participate in the first-price auction sharing the same risk attitude is a convenient assumption. Nonetheless, it is also a very strong and unrealistic assumption, since no matter in the field setting or in the laboratory, it is difficult to control the bidders’ risk preferences. Hence, heterogeneous risk aversion is often believed to be a more realistic assumption.

The papers to first assume that the subjects differ in their risk preferences are by CRS (1982) and CSW (1982) (Constant Relative Risk Aversion, CRRA model). They have quite a large number of observations of bidding in auctions with 3, 4, 5, 6, and 9 bidders with uniform distribution values. Instead of assuming the same risk parameter, they suppose that the bidders’ risk preferences are from some distribution on  $(0, 1]$ . Each bidder knows his own risk parameter, and also knows the distribution. In 1988, Cox, Smith, and Walker formulate a model (log-concave), which includes risk loving preference in the CRRA model. The CRRA model encompasses both the homogeneous

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<sup>26</sup> Harris and Raviv (1981), Holt (1980), Maskin and Riley (1980) and Riley and Samuelson (1981).

and heterogeneous cases for the bidders' risk preferences. In this chapter, however, the terminology 'CRRA' model particularly refers to heterogeneous risk aversion.

In general, the CRRA model fits the experimental data quite well. For example, Palfrey and Pevnitskaya (2008) conduct a first-price auction with an outside option, which directly test the CRRA model. They observed a self-selection effect which will only arise with heterogeneous risk preferences. That is, if there is an outside option with a fixed payoff, the more risk loving bidders would still choose to enter into the auction stage, whereas the more risk averse bidders would choose not to, and claim the fixed payoff instead. Therefore, the bids are lower compared to the first-price private value auction without such an outside option.

At the same time, the CRRA model has also led to a number of challenges. First, the CRRA model fails as a maintained hypothesis even for some subjects in CSW (1988)'s first-price auctions - 21.8% of subjects having bidding functions with significant nonzero intercepts, despite the CRRA model predicting a zero intercept (CSW, 1988, p. 77). Second, Harrison (1989) suggests an alternative way to interpret the experimental results by looking at the expected payoff space instead of observing the bid space as in the CRRA model. He argues that the expected cost of deviating from the risk neutral Nash Equilibrium is quite small (less than \$0.05 at the median, the flat maximum critique).<sup>27</sup> To put it another way, when evaluating the bidding behaviours in terms of the forgone expected payoffs, the deviations from the RNNE model are no longer significant. Third, although risk aversion provides a good fit for some novel treatment conditions, for instance, when private values are drawn from a non-uniform distribution (Chen & Plott,

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<sup>27</sup> The payoff function around the maximum is flat. This also received some follow up criticism (see the detailed survey by Kagel & Roth, 1995): Friedman (1992) pointed out 'a more appropriate payoff space should consider the consequences of all bids, not only those resulting in median expected losses'.

1998),<sup>28</sup> there are still some auction settings where bids contradict the risk aversion hypothesis. These include second-price, third-price, multiunit first-price, and asymmetric auctions. The CRRA model fails to explain the persistent overbidding in second-price auctions (the winner pays the second-highest bid), since the dominant strategy of bidding one's value is immune to risk attitude. The other finding is that in third-price auctions, the risk averse bidders should bid below instead of above the RNNE bidding line according to Constant Absolute Risk Aversion (CARA) model, as in first-price auctions. Kagel and Levin (1993) use a dual market procedure with five and ten bidders. They find that in the ten-bidder market, the majority of bids lie above the RNNE prediction, whereas at the same time, the majority of bids in auctions with five bidders are below the RNNE prediction. This implies that for the same group of subjects their risk attitudes change as the market size varies.

A third example is in the multiple unit discriminate auctions conducted by Cox, Smith and Walker (1984) with N number of bidders competing for Q homogeneous objects. Four out of the ten treatments are characterized by a pronounced tendency of individuals to bid lower than the risk neutral prediction which implies risk-loving behaviour.

A fourth example is asymmetric auctions, which refers to the auctions including strong bidders (whose valuations are likely to be higher) and weak bidders (whose valuations are likely to be lower). According to the theory, the expected revenue for second-price auctions should be higher than first-price auctions in the asymmetric value context.<sup>29</sup> However, in Pezanis-Christou (2002), the average revenue in the first-price auctions is stochastically greater than in the second-price auctions. This result cannot be attributed

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<sup>28</sup> They also comment that the CRRA result cannot rule out the possibility of bidders just using a linear decision rule such as a constant percentage markdown.

<sup>29</sup> This is because bidding one's value is still a dominant strategy in second-price auctions, whereas in first-price auctions the strong bidders place low bids when they get low values.

to risk aversion, as even with extremely risk averse bidders, the revenue difference would not be as much as the data suggest. The bids deviation from RNNE decreasing over time is also contrary to the risk aversion model which indicates that the bidding strategy is a constant proportion of the values. Despite the above shortcomings of the CRRA model, Kagel and Roth (1995, p. 536) summarise that risk aversion may have some role, but not the only role to play in explaining behaviour in private value auctions and risk aversion is very much used in empirical studies of auction data. Thus, we cannot rule out risk aversion as an explanation of overbidding in first-price auctions.

The goal of this chapter is twofold. First, to check how the CRRA model's predictions are affected by the assumptions used to define heterogeneity in first-price auctions. As we know, the CRRA model requires the subject to be aware of not only his own risk parameter, but also the distribution of the risk parameter in the population. In addition, the distribution is common knowledge to all the bidders. This is quite a strong assumption. Naturally we would think the bidders' belief about the risk parameter distribution should matter a lot in estimating the risk aversion bidding function.

The theoretical paper by Van Boening, Rassenti, and Smith (1998) illustrates how changing three assumptions-the maximum propensity to risk seeking  $r_{max}$ , the expected value of bidders' risk preferences  $E(r)$ , and the probability that a subject is risk loving  $p(r > 1)$  would influence the nonlinear part of the bidding function. They use numerical methods to compute the Nash equilibrium bidding function in a four-bidder first-price auction by varying the three elements. They find that the effect of changing  $r_{max}$  from two to three, does not appear to have a substantial effect on the bidding function and the shape of the risk parameter distribution, when  $E(r)$  and  $p(r > 1)$  are held constant. In this chapter, as well as presuming each bidder differs in the risk attitudes, each bidder's risk parameter is estimated in two different assumptions: one is that we limit the risk

aversion parameter in the range of zero to one, which represents that the bidders are risk-averse or risk-neutral; the other is to push the risk aversion parameter boundary to two, so as to allow the possibility of risk loving. Then we compare the distributions within two assumptions. We find that although changing the support for the distribution influences each individual's estimated risk aversion parameter, it does not significantly affect the distribution. Therefore, this chapter provides further evidence for the robustness of the CRRA model. Regardless of whether the bidders hold a relatively stronger common belief that no bidder is risk loving, or they share a more relaxed belief that some bidders are risk loving, actually the resulting distributions of individuals' risk parameters are not substantially different.

In addition, to give a complete idea of the subjects' parameters and the corresponding distribution relating to a payoff uncertainty, analysis of the loss aversion coefficients is undertaken. The data from two groups of subjects participating in the same lottery task is used, which includes 14 lotteries that each subject must choose to 'Accept' or 'Reject'. The first group is the 2016 experiment, which we explain and analyse in Chapter 1. The second group is from the 2014 experiment conducted by Pezanis-Christou and Wu (2014). After estimating the subjects' loss aversion coefficient using interval regression technique and plotting the corresponding two groups' distributions of the loss aversion parameters, we find that the two distributions are stochastically equivalent. Such a result, together with the results from the risk aversion parameters elicited in the first-price auction stage, suggest that the subjects' parameters relating to risk are heterogeneous but quite stable within the same experimental institution with regards to the overall distribution.

Another aim of this chapter is to identify the treatment effect in the unusual first-price auction design by CRS (1982). A standard first-price auction experiment consists of several auction rounds. Prior to each auction round, each subject's private value is



determined randomly from the same distribution. Each subject's private values in different auction rounds are independent. Different from the standard first-price auction, in CRS (1982)'s design the subjects were given all the private values they would have in the future auction rounds. Pezanis-Christou and Romeu (2006) pointed out that the experimental procedure used by CRS (1982) used to collect the data may have interfered with the outcomes. Nonetheless, so far there is no research on how the 'all private values shown before the auction starts' design could impact the bidding behaviour compared to the standard first-price auction design. Here, we observe that the bids are significantly lower in the design by CRS (1982) compared to the standard first-price auction experiment.

This chapter is structured as follows. The next section presents the theoretical CRRA model of bidding with independent private values in first-price auction. Thereafter the equilibrium prediction is set out. Section 2.3 describes the data. The treatment effect and risk parameter estimation results, as well as the loss aversion coefficients results are presented in Section 2.4. Finally, Section 2.5 summarises and concludes.

## 2.2 Model

Consider  $n$  bidders participating in a first-price sealed-bid auction. They compete for a single object, which is awarded to the highest bidder for a price equal to his bid. Each bidder  $i = \{1, 2, \dots, n\}$  has an induced value  $v_i$  which is an independent draw from a uniform distribution  $F$  with support  $[0, 1]$  and density  $f$ . To present the CRRA model, we start from the equilibrium model with homogeneous risk averse bidders who share the same risk parameter  $r$ . In Chapter 1, we show that the risk averse symmetric Nash equilibrium (RASNE) bidding function is as follows

$$b(v_i)^{RASNE} = \frac{n-1}{n+r-1} v_i \quad (2.2.1)$$

However, since homogeneous risk aversion is a very restrictive assumption, CSW (1988) extended the model by assuming that the individual risk parameter  $r_i$  is independently drawn from a continuous distribution  $\Phi$  over the interval  $(0, r_{max}]$ , where  $r_{max}$  stands for the least risk averse bidder's risk parameter. Each bidder knows his own risk averse parameter  $r_i$ , but only knows the others' distribution  $\Phi$ . In addition,  $\Phi$  is common knowledge to all the bidders. When  $0 < r_i < 1$  then bidder  $i$  is risk averse; if  $r_i = 1$  then he is risk neutral; and if  $r_i > 1$  he is risk loving. Therefore, equation (2.2.1) becomes

$$b(v_i)^{CRRRA} = \frac{n-1}{n+r_i-1} v_i \quad (2.2.2)$$

We need to emphasise that such a pure strategy equilibrium is not fully established. It only holds for bids that do not exceed  $b^*$  (CRS, 1982), where  $b^*$  is the maximum bid that would theoretically be entered by the least risk averse subject in the population; and there is no closed form solution for bids over  $b^*$ .<sup>30</sup> Therefore, it is difficult to verify whether the bids are monotonic over the whole range of private values.

$$b^* = \frac{n-1}{n-1+r_{max}} \quad (2.2.3)$$

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<sup>30</sup> This is because for these bids we cannot disentangle the two reasons for the high bids: high private values or very risk-averse attitudes.

To address this problem, Van Boening, Rassenti, and Smith (1998) numerically solve the nonlinear part of the bid function in a 4-bidder market with uniformly distributed private values. With regards to the distribution  $\Phi$  of the risk preference parameters, they assume that it is a beta distribution on  $[0, r_{max}]$  by verifying the values of the most risk loving bidder's risk parameter in three cases:  $r_{max} = 1, 2$ , and  $3$ , respectively. Furthermore, in determining the nonlinear part of the bid function, they assume the existence of two additional parameters that do not enter into the linear part of the function. They are  $E(r_i)$  and  $P(r_i > 1)$ , which are the expected value of the risk parameters and the probability that any bidder is risk loving, respectively. As a result, they verify the nonlinear property of the bid function above  $b^*$ . In addition, they find that increasing  $r_{max}$  from 2 to 3 - while holding  $E(r_i)$  and  $P(r_i > 1)$  constant, does not change the estimated bid function significantly.

## 2.3 Data

The data we use in this chapter is originally from 47 experiments executed variously by CRS (1982), CSW (1983), and CSW (1985). CSW (1988) also summarise the important characteristics of these experiments.

The CRS (1982) experiments consist of 3 switchover sequences with 3, 4, 5, 6, and 9 bidders: 10 first-price (Dutch) auctions, followed by 10 Dutch (FP) auctions, and finally another 10 first-price (Dutch) auctions. Since Dutch auctions are beyond our scope, we only illustrate how the first-price auction is conducted: in each round of the auction, the private values are drawn in multiples of \$0.1 from a uniform distribution  $[\underline{v}, \bar{v}]$  with  $\underline{v} = \$0.1$ . Instead of giving the subject's induced value at the outset of each round of the auction, there is a spread sheet that discloses the subject's private values in all the 10

rounds, which is quite an unusual design. Across the different market sizes, in order to keep the subjects' (as risk neutral bidders) expected earnings approximately constant, CRS (1982) deliberately set  $\bar{v}$  to increase as market size increases.<sup>31</sup> The bidder can submit a bid up to his private value. The design of two experiments with 4 and 5 bidders by CSW (1983) followed the same procedure as CRS (1982), only with triple the payoff levels.<sup>32</sup>

Compared with the above two groups of experiments, the first-price auction design in CSW (1985) with 3 and 4 bidders is quite standard in the sense that the subject only knows his private value in the current round. The private values are drawn in multiples of \$0.01 from a uniform distribution  $[\underline{v}, \bar{v}]$  with  $\underline{v} = 0$ . Besides this, they have no information about how many auction rounds he will participate in. Furthermore, the bidder can bid up to  $\bar{v}$ , which is the highest possible valuation instead of up to his private value.

The information feedback is the same for all the treatments: at the end of each round, the subject finds out what the winning bid is. The final payoff for each subject is the accumulated payoff across all the auction rounds. The protocol details of the experiments are summarised in Table 2.3.1.

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<sup>31</sup> It permits some consistency over motivation since the theory predicts that the expected earning approaches zero as  $N$  becomes larger. However, this design receives some criticism from Kagel and Roth (1992, p. 1388). They think it should be designed in a way that either the valuation distribution or the number of bidders is held constant.

<sup>32</sup> The reason for designing the tripled payoff experiments is to test whether multiplying the profit of a winning bid by any factor could have any effect on the equilibrium bid. The result for the experiment shows that there is no impact on bids, which is consistent with the CRRA model. However, Kagel (1995, p. 527) suggests that the experiments require substantial adjustments in order to provide a more demanding test.

Table 2.3.1 Details of first-price experiment data

| Paper      | Series | Sessions                  | $[\underline{v}, \overline{v}]$ | # Rounds | #Observations |
|------------|--------|---------------------------|---------------------------------|----------|---------------|
| CRS (1982) | 1(3)   | dfd10'; dfd10x; dfd3      | [0.1, 4.9]                      | 10       | 90            |
|            |        | fdf10; fdf3'              |                                 | 20       | 120           |
| CSW(1985)  | 1'(3)  | fpn3(1); fpn3(2)          | [0, 6.0]                        | 20       | 300           |
|            |        | fpn3(1)x,... fpn3(3)x     |                                 |          |               |
| CRS (1982) | 2(4)   | dfd8'; dfd8x              | [0.1, 8.1]                      | 10       | 80            |
|            |        | fdf8; fdf8'x              |                                 | 20       | 160           |
| CSW(1983)  | 3(4)   | dfd8x*                    | [0.1, 8.1]                      | 10       | 40            |
|            |        | fdf8'x*                   |                                 | 20       | 80            |
| CSW(1985)  | 4(4)   | fplonci1,... fplonci10    | [0, 10.0]                       | 25       | 1000          |
|            | 9(4)   | fp1nt1(1)x                | [0, 10.0]                       | 12       | 48            |
|            |        | fp1nt1(2)x; fp1nt1(3)x    |                                 | 20       | 160           |
|            | 10(4)  | fpbasei(1),... fpbasei(3) | [0, 10.0]                       | 25       | 300           |
|            | 11(4)  | fpbnt(1)x,... fpbnt(3)x   | [0, 10.0]                       | 20       | 240           |
| CRS (1982) | 5(5)   | dfd9; dfd9'x              | [0.1, 12.1]                     | 10       | 100           |
|            |        | fdf9'; fdf9x              |                                 | 20       | 200           |
| CSW(1983)  | 6(5)   | dfd9*                     | [0.1, 12.1]                     | 10       | 50            |
|            |        | fdf9'*                    |                                 | 20       | 100           |

|            |      |              |             |    |     |
|------------|------|--------------|-------------|----|-----|
| CRS (1982) | 7(6) | dfd2ri; dfd4 | [0.1, 16.9] | 10 | 120 |
|            |      | fdf2'; fdf4' |             | 20 | 240 |
|            |      |              |             |    |     |
| CRS (1982) | 8(9) | dfd5         | [0.1, 36.1] | 10 | 90  |
|            |      | fdf5'        |             | 20 | 180 |

*Note: This table is compiled from Table 1 in CRS (1982) and CSW (1988), respectively. The letter 'X' represents the sessions with experienced subjects; \* indicates the sessions with tripled payoffs. The numbers in the brackets of column 'Series' stand for the market sizes.*

## 2.4 Analyses

### 2.4.1 Treatment Effect

It is noticed that apart from the differences in market size and payoff structure, another major disparity between the experiments that needs to be considered is the availability of information about private values in future rounds. In CRS (1982) experiment, each subject knew in advance their private values for all 10 rounds of a given first-price auction segment. Therefore, in this section, we analyse whether this design would have any influence on bidding behaviour. For simplicity, we refer to the CRS (1982) experiment as the 'Knowledge treatment', and the CSW (1985) experiment as the 'No-Knowledge treatment'.<sup>33</sup> We consider the two treatments, as well as the two different market sizes (3-bidder and 4-bidder) as a 2 X 2 full-factorial design.<sup>34</sup> We summarise the experimental design in Table 2.4.1. There are two treatments: 'No-Knowledge' versus 'Knowledge'

<sup>33</sup> CSW (1983) also conducted the Knowledge treatment, however it gave the subjects tripled payoff instead. Therefore, we do not analyse CSW (1983) in this section.

<sup>34</sup> One limitation of the between-subject design is that we cannot control for the individual differences.

(with ‘No-Knowledge’ treated as the ‘control’); and ‘n=3’ versus ‘n=4’ (with ‘n=3’ treated as the ‘control’).

Table 2.4.1 The No-Knowledge treatment and Knowledge treatment

|              | n=3                     | n=4                      |
|--------------|-------------------------|--------------------------|
| No-Knowledge | 2 sessions; 6 subjects  | 10 sessions; 40 subjects |
| Knowledge    | 4 sessions; 12 subjects | 2 sessions; 8 subjects   |

*Note: We remove the sessions with experienced bidders.*

Figure 2.4.1 illustrates the actual bids and risk neutral Nash equilibrium (RNNE) bid predictions for the above four treatments, and we find that most bids fall into the risk aversion Nash Equilibrium domain.<sup>35</sup> At the same time, in the 3-bidder market, the bids are lower in the Knowledge treatment relative to the No-Knowledge treatment, on average. As can be seen from the top panel of the 3-bidder Knowledge treatment, 78 out of 180 observations (43.3%) are below the RNNE prediction. However, such a difference is not significant in the 4-bidder market.

<sup>35</sup> In the n=4 No-Knowledge treatment, 34 out of 1000 bids are above the 45 line, which is irrational since this could result in loss if they are the winning bids. CSW (1988, p. 78) describe this behaviour pattern as ‘playing Russian roulette’.

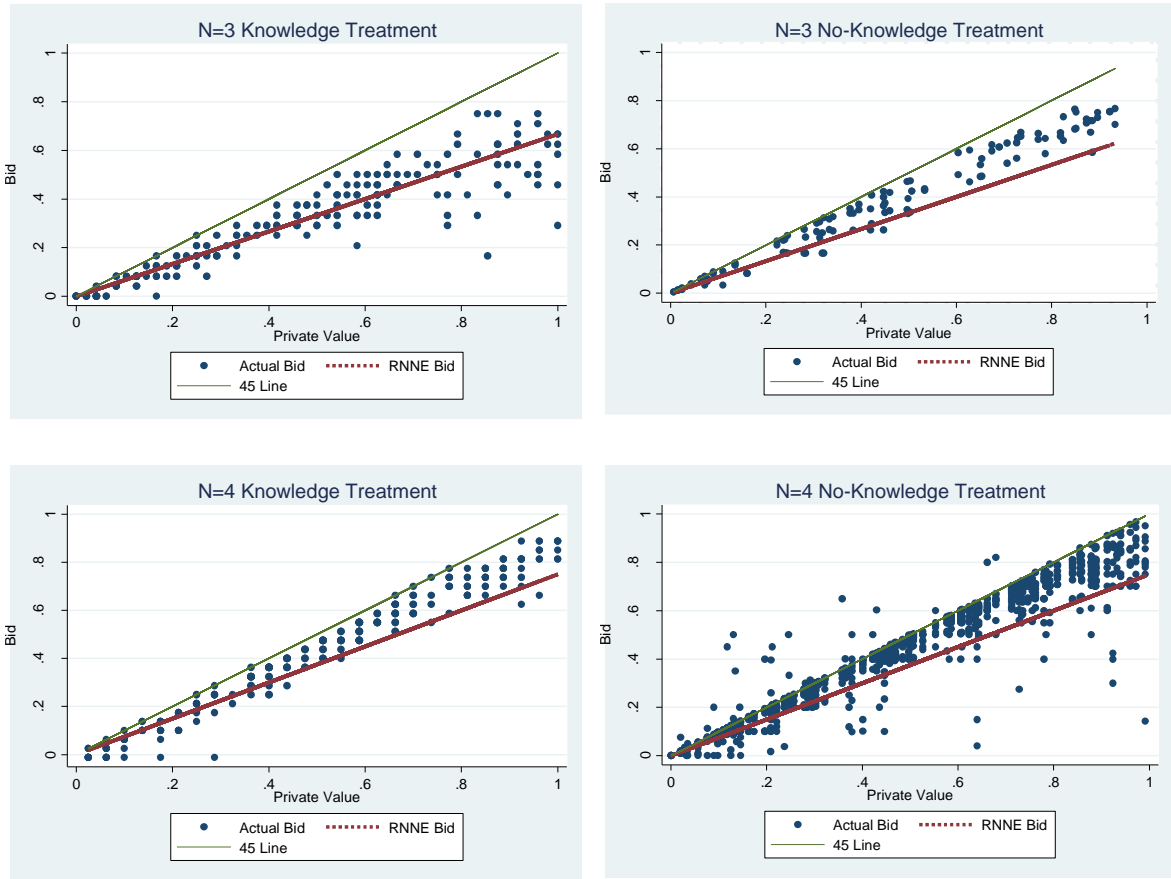


Figure 2.4.1 Bids and private values in the Knowledge and No-Knowledge treatments for N=3 and N=4

After removing 34 zero bid observations, we estimate an econometric model to test the effect of the Knowledge treatment on bids while holding other relevant factors constant. Note that in the corresponding experimental between-subject design the treatment variables are constant across auction rounds. Therefore, a pooled OLS model is employed. To control for market size effects, the model includes a dummy variable for market size as well as interaction terms with the main treatment variable and the private value. We use the following specification



$$bid_{it} = \beta_0 + \beta_1 pv_{it} + \beta_2 Know_i + \beta_3 (pv_{it} \times Know_i) + \beta_4 Msize_i + \beta_5 (pv_{it} \times Msize_i) + \beta_6 (pv_{it} \times Know_i \times Msize_i) + \varepsilon_{it} \quad (2.4.1)$$

where the dependent variable  $bid_{it}$  denotes the bid of subject  $i$  submitted at auction round  $t$ . A dummy variable for the treatment variable is included as independent variable  $Know_i$ , which equals 1 if the observation belongs to the Knowledge treatment and 0 if it belongs to the No-Knowledge treatment.  $Msize_i$  is also a dummy variable, which equals 1 if the observation takes place in a 4-bidder market and 0 if it takes place in a 3-bidder market. Since in the No-Knowledge treatment, each bidder can bid up to  $\bar{v}$  instead of his private value  $pv_{it}$ , we discard the bid observations where the bid exceeds the private value. In addition, we also normalise the bids and private values on the unit interval.<sup>36</sup> Column (1) in Table 2.4.2 presents the regression coefficients.

As recorded in the first and second rows of column (1) in Table 2.4.2, for the baseline No-Knowledge treatment in the 3-bidder market, the estimated intercept is insignificant, which accords with the CRRA prediction. However, the slope coefficient 0.798 is significantly above the RNNE prediction (0.667), which suggests that the bidders are risk averse on average. Table 2.4.2 also reveals that both  $\beta_2$  and  $\beta_4$  are insignificant, which indicates that the different treatments (No-Knowledge versus Knowledge, and  $n=3$  versus  $n=4$ ) do not influence the intercept term. At the same time, the differences in the slopes are both significant. In the 3-bidder market, the estimated slope in the Knowledge treatment is reduced by 0.145 relative to the No-Knowledge treatment ( $\beta_3$ ), thus results to the estimated slope is 0.653, which does not violate the RNNE prediction. In addition,  $\beta_5$  is significantly positive, which shows that in terms of the No-Knowledge

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<sup>36</sup> Normalisation is achieved as follows:  $\tilde{v} = \frac{v-\underline{v}}{\bar{v}-\underline{v}}$ ,  $\tilde{b} = \frac{b-\underline{v}}{\bar{v}-\underline{v}}$ .

treatment, the bidders submit higher bids in the 4-bidder market on average compared to the counterpart 3-bidder market, and this fits our prediction.

The bidding behaviour for the Knowledge treatment in the 4-bidder market needs to be checked as well. As we can see from Table 2.4.2,  $\beta_6$ , the interaction slope term of the two dummy variables is qualitatively close to  $\beta_3$ . This suggests that the negative impact of the Knowledge treatment on the bids is offset by the market size increasing.

***Result 1: In the 3-bidder market, the bids are lower in the Knowledge treatment relative to the No-Knowledge treatment, on average. However, such a difference does not exist in the 4-bidder market.***

Such results are also in accordance with CRS (1982, p. 25) in that noncooperative behaviour fails to apply in the Knowledge treatment where  $n=3$ . Furthermore, we use piecewise linear regression technique to provide some insights of how the Knowledge treatment influences bids on different private value ranges in the 3-bidder market. Column (2) in Table 2.4.2 reports the results. In the piecewise linear estimation, we separate the normalized private values into four equal segments:  $[0, 0.25)$ ,  $[0.25, 0.5)$ ,  $[0.5, 0.75)$ , and  $[0.75, 1]$ . We find that, the Knowledge treatment pushes down the bids for all four value ranges. However, only in range  $[0.5, 0.75)$  is the coefficient significant (-0.357), which results in a much lower fraction of private value 0.587 in the corresponding private value range compared with the No-Knowledge treatment (0.944). Therefore, we have the following result

***Result 2: In the 3-bidder market, bidders submit significantly lower bids when they receive median private values in the Knowledge treatment relative to the No-Knowledge treatment.***

Table 2.4.2 Pooled OLS regression for the observed bids

| Independent Variable | Dependent variable: bid |                     |
|----------------------|-------------------------|---------------------|
|                      | (1)                     | (2)                 |
| Intercept            | 0.014<br>(-1.53)        | -0.002<br>(-0.33)   |
| PV                   | 0.798***<br>(-46.38)    | -                   |
| 1                    | -                       | 0.802***<br>(16.24) |
| 2                    | -                       | 0.823***<br>(16.28) |
| 3                    | -                       | 0.944***<br>(21.97) |
| 4                    | -                       | 0.529***<br>(5.22)  |
| Know                 | -0.014<br>(-1.61)       | -                   |
| PV X Know            | -0.145***<br>(-5.27)    | -                   |
| 1 X Know             | -                       | -0.071<br>(-0.96)   |

|                   |                     |                     |
|-------------------|---------------------|---------------------|
| 2 X Know          | -                   | -0.045<br>(-0.55)   |
| 3 X Know          | -                   | -0.357**<br>(-3.70) |
| 4 X Know          | -                   | -0.189<br>(-1.17)   |
| Msize             | -0.01<br>(-1.10)    | -                   |
| PV X Msize        | 0.076*<br>(3.56)    | -                   |
| PV X Know X Msize | 0.149***<br>(-4.41) | -                   |
| Adj. $R^2$        | 0.933               | 0.913               |
| # Observations    | 1386                | 300                 |

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*Note: Column (1) reports the regression results for equation (2.4.1). Column (2) reports the piecewise linear regression results for the 3-bidder market. Robust standard errors are shown in parentheses; \*\*\*  $p < 0.001$ , \*\*  $p < 0.01$ , \*  $p < 0.05$ ; '1', '2', '3', and '4' represent the private value range [0, 0.25), [0.25, 0.5), [0.5, 0.75), and [0.75, 1] respectively.*

### 2.4.2 Efficiency in different treatments

After identifying the treatment effect, we check whether different treatments as well as different market sizes would have any impact on the efficiency of allocations.

An auction is efficient if the object has been given to the bidder who values it the most. The efficiency of an auction can be measured as the ratio of the winner's value to the highest value drawn among the bidders.

Table 2.4.3 Mean efficiency by treatment and number of bidders

| N | Paper     | Treatment          | # Auction | # Obs. | Efficiency     |
|---|-----------|--------------------|-----------|--------|----------------|
| 3 | CRS(1982) | Knowledge          | 70        | 77     | 96.2% (97.61%) |
|   | CSW(1985) | No-Knowledge       | 100       | 100    | 99.10%         |
| 4 | CRS(1982) | Knowledge          | 60        | 66     | 98.8% (99.62%) |
|   | CSW(1983) | Knowledge, Tripled | 30        | 36     | 98.80%         |
|   | CSW(1985) | No-Knowledge       | 437       | 439    | 98.60%         |
| 5 | CRS(1982) | Knowledge          | 60        | 64     | 99.5% (99.80%) |
|   | CSW(1983) | Knowledge, Tripled | 30        | 31     | 98.50%         |
| 6 | CRS(1982) | Knowledge          | 60        | 63     | 98.5% (98.26%) |
| 9 | CRS(1982) | Knowledge          | 30        | 33     | 99.6% (99.77%) |

*Note: For each combination of treatment and market size, we combine all the associated auctions and observations to compute the mean allocation efficiency. For the rows where '# Obs' outweighs '# Auction', more than one bidder submitted the highest bid in the corresponding auctions. The numbers in parentheses indicate the mean efficiency as calculated in CRS (1982), Table 9.*

In Table 2.4.3, we report the mean efficiency by treatment and number of bidders. CRS (1982, p. 28) also report the mean efficiency for the CRS (1982) experiments (as shown in parentheses in the last column of Table 2.4.3). There are some differences between our results and the efficiencies reported by CRS (1982). This is because for some auction rounds, more than one bidder submitted the highest bid and only CRS (1982) know which bidder was selected as the winner. However, in this section, we extend the efficiency calculation by incorporating CSW (1983) and CSW (1985) experiments and thus provide a full picture of the allocation efficiency in different treatments. It is observed that the efficiencies are quite high in all the experiments (except that the efficiency of the Knowledge treatment in the 3-bidder market is slightly lower).

### 2.4.3 Heterogeneous risk preferences

In Section 2.2, we have shown that the equilibrium bid function for the CRRA model is:

$$b_i = \frac{n-1}{n-1+r_i} v_i \quad (2.4.2)$$

for 
$$b_i \leq b^* = \frac{n-1}{n-1+r_{max}} \quad (2.4.3)$$

Where  $b^*$  is the maximum bid for the least risk averse bidder in the population.<sup>37</sup> The part of the equilibrium bid function for  $b_i > b^*$  has no closed form. Equation (2.4.2) is dependent on the risk attitude of bidder  $i$  but not of his rivals.

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<sup>37</sup> We have normalized the private values in the unit interval.

The CRRA model assumes that bidders behave as if a certain distribution over risk attitudes is commonly known by them, as in CSW (1988, p. 75) and Walker, Smith and Cox (1990, p. 12). These papers set  $r_{max} = 1$ ; put it another way, the bidders share the belief that the least risk averse bidder is risk neutral. In this section, we relax this assumption so that risk-loving bidders exist in the population, and set  $r_{max} = 2$ .<sup>38</sup> Therefore, we can estimate each bidder's risk preference parameter  $r_i$  within these two assumptions.

Harrison (1990) notices an issue with testing the CRRA model, which is that laboratory experiments generally do not control for the bidders' risk attitudes. Therefore, he uses the first-price auctions dataset from CRS (1982) and CSW (1983) and constructs four explicit Bayesian prior distributions for the risk attitudes. Harrison identifies that, if the risk-loving propensity is ignored and 100 values for  $r_i$  (from 0.01 to 1.00) are used to generate the explicit prior, the winning bids are consistent with the risk neutral Nash predictions.

However, when one allows risk-loving bidders, they are inconsistent with the Nash predictions. Overall, there are two major differences between Harrison (1990) and this chapter. Firstly, after assuming the prior distribution, the method employed by Harrison is to generate the predicted winning prices using equation (2.4.2), and compare them with the realised winning price. In this chapter, we use the realised bids, not only the winning bids, to back out  $r_i$  using equation (2.4.2) and to compare how changing  $r_{max}$  would influence  $r_i$  and the corresponding underlying distributions. Secondly, with regards to allowing for risk-loving bidders, Harrison does not confine  $r_{max}$ , whereas in this chapter we set  $r_{max} = 2$ .

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<sup>38</sup> Referring to Chapter 1,  $r_{max} = 2$  is already in the range of highly risk loving. Therefore, setting such a prior belief for the least risk averse bidder is sufficient and we do not need to raise the assumption of  $r_{max}$  to an even higher value.

We analyse each bidder's bidding behaviour by estimating the following linear equation

$$b_i = \alpha_i + \beta_i v_i + e_i \quad (2.4.4)$$

Where  $b_i$  and  $v_i$  are normalised to the unit interval, and  $e_i$  is a random error term with mean zero. This linear equation is only valid for the observations which satisfy equation (2.4.3).<sup>39</sup> For a given number of bidders,  $b^*$  is inversely related to  $r_{max}$ , so that there would be fewer valid observations as  $r_{max}$  increases. From the CRRA model we have the prediction that  $\alpha_i = 0$  and  $\beta_i = \frac{n-1}{n-1+r_i}$ . When estimating bidder  $i$ 's coefficient pair  $(\hat{\alpha}_i, \hat{\beta}_i)$ , if  $\hat{\alpha}_i$  is significantly different from zero (either positive or negative), then bidder  $i$ 's bidding behaviour is inconsistent with the CRRA model. Otherwise, CRRA model implies that

$$\hat{r}_i = \frac{(1 - \hat{\beta}_i)(n - 1)}{\hat{\beta}_i} \quad (2.4.5)$$

We discard from the analysis the few sessions that involve subjects with experience from previous auction experiments, and also discard the few sessions with tripled payoffs.<sup>40</sup> The data we use constitutes a total of 2500 observations, spread over 29 sessions and a summary of the regression estimates  $(\hat{\alpha}_i, \hat{\beta}_i)$  for Equation (2.4.4) are reported in Table 2.4.4.

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<sup>39</sup> CSW (1988, p.75); Walker, Smith, and Cox (1990, p.12); Cox, Smith, and Walker (1992, p. 1400), and Pezanis-Christou and Romeu (2002, p.13) used a more demanding iterative method to truncate the observations where  $v_i > v^* = \frac{n-1+r_i}{n-1+r_{max}}$ .

<sup>40</sup> As mentioned in footnote 3 of Harrison (1990), there is some behavioural evidence that the experienced subjects are more likely to be classified as risk averse or risk loving instead of risk neutral.



Table 2.4.4 Summary of regressions estimates of  $\hat{\alpha}_i, \hat{\beta}_i$  in the cases of  $r_{max} = 1$  and  $r_{max} = 2$

| N     | Treatment    | # Subjects. | # Total Auctions | $r_{max} = 1$   |                                      |                      |                 | $r_{max} = 2$                        |                      |                      |                 |
|-------|--------------|-------------|------------------|-----------------|--------------------------------------|----------------------|-----------------|--------------------------------------|----------------------|----------------------|-----------------|
|       |              |             |                  | # Obs (Percent) | $\hat{\alpha}_l \neq 0$ <sup>a</sup> |                      | # Obs (Percent) | $\hat{\alpha}_l \neq 0$ <sup>b</sup> |                      |                      |                 |
|       |              |             |                  |                 | $\hat{\alpha}_l > 0$                 | $\hat{\alpha}_l < 0$ |                 | Total                                | $\hat{\alpha}_l > 0$ | $\hat{\alpha}_l < 0$ | Total           |
| 3     | Knowledge    | 12          | 180              | 151 (83.89)     | 0                                    | 0                    | 0 <sup>c1</sup> | 125 (69.44)                          | 0                    | 0                    | 0 <sup>c1</sup> |
|       | No-Knowledge | 6           | 120              | 103 (85.83)     | 0                                    | 1                    | 1               | 80 (66.67)                           | 0                    | 1                    | 1               |
| 4     | Knowledge    | 8           | 120              | 99 (82.50)      | 0                                    | 3                    | 3               | 82 (68.33)                           | 0                    | 3                    | 3               |
|       | No-Knowledge | 52          | 1300             | 1076 (82.77)    | 8                                    | 3                    | 11              | 836 (64.31)                          | 2                    | 5                    | 7 <sup>c2</sup> |
| 5     | Knowledge    | 10          | 150              | 120 (80.00)     | 0                                    | 3                    | 3               | 93 (62.00)                           | 0                    | 1                    | 1               |
| 6     | Knowledge    | 24          | 360              | 315 (87.50)     | 0                                    | 3                    | 3               | 284 (78.89)                          | 0                    | 2                    | 2               |
| 9     | Knowledge    | 18          | 270              | 243 (90.00)     | 0                                    | 8                    | 8               | 222 (82.22)                          | 0                    | 10                   | 10              |
| Total |              | 130         | 2500             | 2107 (84.28)    | 8                                    | 21                   | 29              | 1722 (68.88)                         | 2                    | 22                   | 24              |

Note: a, b: Number of subjects where  $\hat{\alpha}_i$  is significantly different from zero at the 5% level. c1 (c2): In the corresponding treatments, two subjects' (one subject's)  $\hat{\beta}_i$  are (is) not significantly different from zero at the 5% level, which is also inconsistent with the CRRA model. The estimation results are obtained after normalizing the bid and private value observations at unit interval. The regressions do not include those observations for which  $b_i > b^* = \frac{n-1}{n-1+r_{max}}$ , the nonlinear domain of the normalized bid function; and do not include observations where  $b_i = 0$  or  $b_i \geq v_i$ .

According to Table 2.4.4, across all the sessions, we remove 15.72% of observations before estimating equation (2.4.4) in the case of  $r_{max} = 1$ ; whereas when  $r_{max} = 2$  we have to delete 31.12% of observations.<sup>41</sup> This accords with what we explain above: there are fewer observations as  $r_{max}$  increases. In the two cases where  $r_{max} = 1$  and  $r_{max} = 2$ , there are 29 subjects (22.31%) and 24 subjects (18.46%) respectively, whose bidding behaviour is inconsistent with the CRRA model, with the subjects' intercepts being significantly different from zero. More specifically, the intercepts show a strong tendency to be negative. In CSW (1988, p. 75) Table 4, they report the regression results for 156 subjects (which includes 33 experienced subjects) in the case of  $r_{max} = 1$ . They find a similar result that 21.8% of subjects' intercepts are significantly different from zero.

Table 2.4.5 shows the means, standard deviations and the medians of the estimated risk parameter  $r$  in each treatment. We can observe that, regardless of the  $r_{max}$  assumption, the subjects are risk averse on average, except in the Knowledge treatment with a 3-bidder market. This result is also in accordance with CSW (1988, pp. 72-73). However, as we discuss in Section 2.4.1, this discrepancy may be due to the treatment effect rather than the subjects having risk-loving tendencies. The estimates also suggest heterogeneity across subjects in terms of risk aversion, which is consistent with CSW (1988). For the estimated risk parameter for each subject, please see the results in Appendix C.

From the last row of Table 2.4.5, we find that the pooled means for the risk parameter for  $r_{max} = 1$  and  $r_{max} = 2$  are quite close: 0.567 and 0.598, respectively. It is instructive to compare the results reported by Harrison (1990, p. 543), which elicits risk attitudes using the Becker-DeGroot-Marshak (BDM) procedure. He computes the pooled means of risk

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<sup>41</sup> Using the method of truncating the observations in which  $v_i > v^* = \frac{n-1+r_i}{n-1+r_{max}}$  results in deleting fewer observations than we do in this chapter, such as in Walker, Smith, and Cox (1990), who remove 5.7% of observations in the 4-bidder market; and in Pezanis-Christou and Romeu (2002), who remove 3% of observations in the 3-bidder and 5-bidder markets.

coefficients by varying  $r_{max}$ . One scenario is to set  $r_{max} = 1$ ; the other scenario is allowing risk loving preferences  $r_{max} > 0$  while not giving a specific number for  $r_{max}$ . The pooled means are 0.698 and 1.051, respectively. Combining Harrison's finding as well as our results, changing the prior distribution of the risk parameter by varying  $r_{max}$ , indeed change the estimated risk parameters for the subjects.

Besides eliciting the estimated risk parameter for each individual, we are also interested in the distribution of the risk parameters in the two cases  $r_{max} = 1$  and  $r_{max} = 2$ . Therefore, we calculate the mean risk parameter in each session and obtain 29 independent observation pairs  $(r1, r2)$  which we use to conduct the Wilcoxon signed rank test. The null hypothesis  $H_0$  of the test is that  $r1$  and  $r2$  are from populations with the same distribution. The p-value (0.105) shows that we cannot reject  $H_0$ . Therefore, the two distributions of the risk parameters under the two conditions  $r_{max} = 1$  and  $r_{max} = 2$  are not significantly different.

***Result 3: Changing the prior distribution of the risk parameter by varying  $r_{max}$  from 1 to 2 indeed changes the estimated individual risk parameters. However, the two distributions of risk parameters are not significantly different.***

Table 2.4.5 The statistics of mean and median risk preference parameters in each treatment

| N     | Treatment    | # Total<br>Subjects | $r_{max} = 1$           |       |       |        | $r_{max} = 2$           |       |       |        |
|-------|--------------|---------------------|-------------------------|-------|-------|--------|-------------------------|-------|-------|--------|
|       |              |                     | # Subjects<br>(Percent) | $r_i$ |       |        | # Subjects<br>(Percent) | $r_i$ |       |        |
|       |              |                     |                         | Mean  | S.D.  | Median |                         | Mean  | S.D.  | Median |
| 3     | Knowledge    | 12                  | 10 (83.33)              | 1.13  | 0.546 | 1.03   | 10 (83.33)              | 1.3   | 0.579 | 1.1    |
|       | No-Knowledge | 6                   | 5 (83.33)               | 0.41  | 0.195 | 0.29   | 5 (83.33)               | 0.52  | 0.289 | 0.61   |
| 4     | Knowledge    | 8                   | 5 (62.5)                | 0.26  | 0.071 | 0.29   | 5 (62.5)                | 0.30  | 0.069 | 0.29   |
|       | No-Knowledge | 52                  | 41 (78.85)              | 0.48  | 0.576 | 0.33   | 44 (84.62)              | 0.52  | 0.755 | 0.26   |
| 5     | Knowledge    | 10                  | 7 (70)                  | 0.41  | 0.109 | 0.37   | 9 (90)                  | 0.28  | 0.355 | 0.32   |
| 6     | Knowledge    | 24                  | 21 (87.5)               | 0.75  | 0.600 | 0.64   | 22 (91.67)              | 0.83  | 0.776 | 0.58   |
| 9     | Knowledge    | 18                  | 10 (55.56)              | 0.31  | 0.191 | 0.29   | 8 (44.44)               | 0.25  | 0.180 | 0.29   |
| Total |              | 130                 | 99 (76.15)              | 0.567 | 0.552 | 0.367  | 103 (79.23)             | 0.598 | 0.706 | 0.405  |

Note: S.D.: standard deviation.

Table 2.4.6 The mean of the risk parameter for each session in the cases of  $r_{max} = 1$  and  $r_{max} = 2$

| N | Treatment    | Sessions   | $r1$  | $r2$  |
|---|--------------|------------|-------|-------|
| 3 | Knowledge    | dfd10'     | 0.820 | 1.080 |
|   |              | fdf10      | 0.810 | 0.850 |
|   |              | dfd3       | 2.338 | 2.338 |
|   |              | fdf3'      | 1.360 | 1.640 |
|   | No-Knowledge | fpn3(1)    | 0.500 | 0.610 |
|   |              | fpn3(2)    | 0.270 | 0.380 |
| 4 | Knowledge    | dfd8'      | 0.250 | 0.316 |
|   |              | fdf8       | 0.289 | 0.405 |
|   | No-Knowledge | fplonci1   | 0.332 | 0.330 |
|   |              | fplonci2   | 0.517 | 0.103 |
|   |              | fplonci3   | 0.525 | 0.554 |
|   |              | fplonci4   | 1.119 | 1.018 |
|   |              | fplonci5   | 0.255 | 0.097 |
|   |              | fplonci6   | 0.688 | 0.804 |
|   |              | fplonci7   | 0.226 | 0.310 |
|   |              | fplonci8   | 0.744 | 1.084 |
|   |              | fplonci9   | 0.300 | 0.363 |
|   |              | fplonci10  | 0.382 | 0.500 |
|   |              | fpbasei(1) | 0.296 | 0.470 |
|   |              | fpbasei(2) | 0.646 | 0.696 |
|   |              | fpbasei(3) | 0.242 | 0.248 |
| 5 | Knowledge    | dfd9       | 0.361 | 0.192 |
|   |              | fdf9'      | 0.474 | 0.395 |
| 6 | Knowledge    | dfd2ri     | 1.119 | 1.199 |
|   |              | dfd4       | 0.686 | 0.759 |
|   |              | fdf2'      | 0.417 | 0.349 |
|   |              | fdf4'      | 0.779 | 0.818 |
| 9 | Knowledge    | dfd5       | 0.389 | 0.285 |
|   |              | fdf5'      | 0.224 | 0.182 |

*Note: The columns of  $r1$  and  $r2$  are obtained by taking the average estimated risk parameters for each subject in the corresponding session with the conditions of  $r_{max} = 1$  and  $r_{max} = 2$ , respectively.*

Besides the statistical test, in order to graphically present the distributions across different treatments and market sizes, we also plot the probability density estimates of  $r1$  (with a red line) and  $r2$  (with a blue line) in Figure 2.4.2. For the sake of comparing the distributions in the same scale, we have controlled the range of the x-axis in the range of  $[0, 2]$  and the y-axis in the range of  $[0, 4]$ . It is obvious that the two unimodal distributions in the Knowledge treatment with 3-bidder are quite different from the other six experiments, as we can see that the local maximum values are in the range of risk neutral or risk loving instead of risk averse. At the same time, the shapes of the distributions of  $r1$  and  $r2$  for the remaining graphs are basically all positively skewed, which shows that the mass of the risk parameter distribution is concentrated to the left of the figure (indicating risk aversion).

After discussing the relationship between the distributions of  $r1$  and  $r2$ , we address another question – is the distribution of the estimated risk parameters the same across different market sizes? To do so, we use the mean values for each session shown in Table 2.4.6 again for the Kruskal-Wallis equality-of-populations rank test. The null hypothesis  $H_0$  of the test is that the  $k$  independent samples come from the same distribution. As previously examined, the subjects in the Knowledge treatment with a 3-bidder market on average are not risk averse, which is different from the other six experiments. Therefore, we remove the corresponding mean values. The results show that no matter whether we use  $r1$  or  $r2$ , we cannot reject that the risk parameters are from the same distribution (p-value = 0.221 and 0.167, respectively). It is instructive to look at a similar result reported by Chen and Plott (1998). They show that the risk parameters estimated from five of the six sessions' first-price auctions with non-uniformly distributed private values are also drawn from the same distribution.

***Result 4: In the CRS (1982) and CSW (1985) experiments with 3, 4, 5, 6, and 9 bidders, the subjects have heterogeneous risk aversion parameters. However, the distributions of the average risk parameters are not stochastically different.***

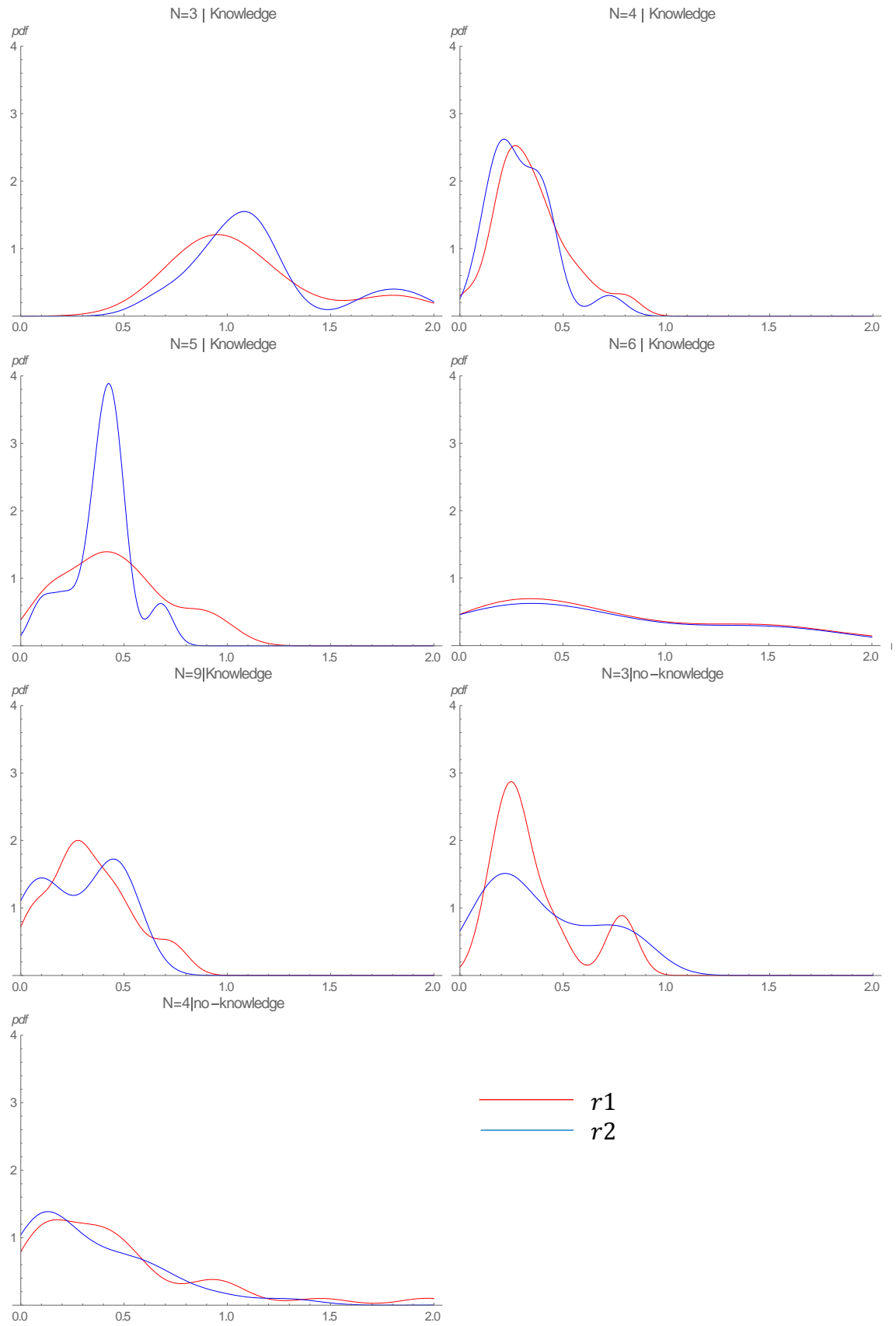


Figure 2.4.2 Probability density estimation with the assumptions  $r_{max} = 1$  and  $r_{max} = 2$  for different market sizes and treatments



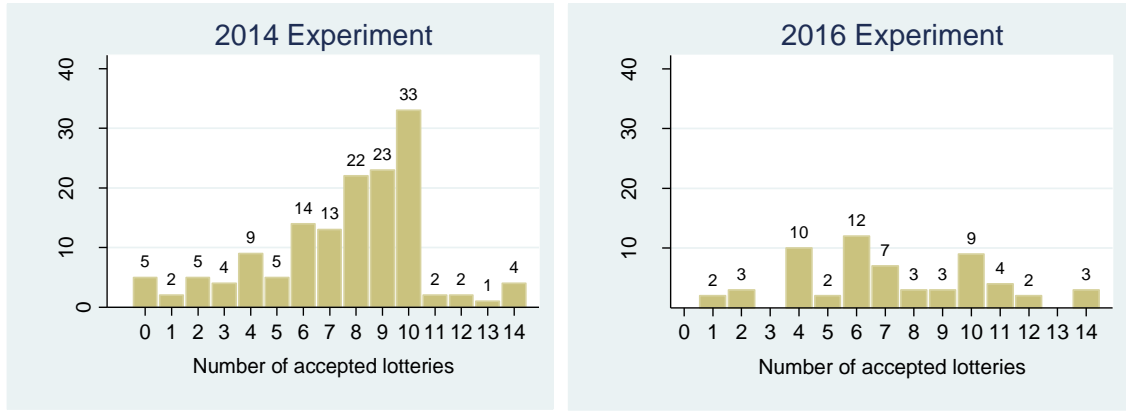
#### **2.4.4 Heterogeneous loss attitudes**

In this section, we analyse loss aversion attitudes, in terms of its importance in economics and particularly in relation to decision-making under uncertainty, which is second only to risk aversion attitudes. It will provide us with a full picture of the subjects' parameters relating to payoff uncertainty. We first describe the data, and then as in Chapter 1 we use an interval regression technique to estimate each subject's loss aversion coefficient based on the data. The last step is comparing the distributions of the loss aversion coefficients between the independent samples we have obtained.

The data we use are from two identical lottery experiments aiming to elicit subjects' loss attitudes. The two experiments include 144 and 60 subjects, respectively. Both experiments were conducted at the Adelaide Laboratory for Experimental Economics (Adlab). One was conducted in 2014, which is from the lottery stage of the working paper -"Loss aversion and regret in common value auctions" by Pezanis-Christou and Wu (2014); the other was conducted in 2016.<sup>42</sup> The design of the 14 lotteries used in the two experiments is shown in Table 1.3.1 in Chapter 1.

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<sup>42</sup> The 2016 experiment is also reported in Chapter 1.



*Note: As in Chapter 1, for the 2014 experiment we also analyse the first switch point for the 21 subjects who have more than one switch points.*

Figure 2.4.3 Distribution of the number of accepted lotteries in the 2014 and 2016 experiments

We first draw histograms of the number of accepted lotteries for the 2014 and 2016 experiments in Figure 2.4.3. As can be seen, for the 2014 experiment, the distribution of the number of accepted lotteries is unimodal and negatively skewed. There is a clear mode and also a most frequent number of accepted lotteries - 10 lotteries, which represents the lotteries with nonnegative expected payoffs. At the same time, the corresponding shape of the distribution in the 2016 experiment is a multimodal distribution with 4, 6, and 10 lotteries as local maximums.

Next, after pooling the observations from the two experiments, we use the interval regression method as in Chapter 1 to elicit each subject's loss aversion parameter. We report the estimated loss aversion coefficient by number of accepted lotteries in Table 2.4.7.<sup>43</sup> The subjects most frequently accepted lotteries #1 to #10 (20.59%). From the fourth column (cumulative percentage) we can find that the median subject of the 204

<sup>43</sup> Due to the number of subjects available for analysis expanding to 204, the estimated loss aversion parameters  $\lambda$  for the corresponding number of accepted lotteries: 1, 2, 4 and 13 are slightly different from what we report in Chapter 1.

subjects accepts lotteries #1 to #8, which implies that the median value of  $\lambda$  is 1.36. This suggests that loss aversion is a significant pattern for the subjects.

Table 2.4.7 The estimated loss aversion parameter for the corresponding number of accepted lotteries

| # Accepted<br>Lotteries | # Subjects | Percentage<br>(%) | Cum.<br>percentage (%) | $[\lambda_{min}, \lambda_{max}]$ | $\lambda$ |
|-------------------------|------------|-------------------|------------------------|----------------------------------|-----------|
| 0                       | 5          | 2.45              | 1.20                   | $(19, \infty)$                   | 19.61     |
| 1                       | 4          | 1.96              | 4.41                   | $(9, 19]$                        | 10.22     |
| 2                       | 8          | 3.92              | 8.33                   | $(5.67, 9]$                      | 6.94      |
| 3                       | 4          | 1.96              | 10.29                  | $(4, 5.67]$                      | 4.78      |
| 4                       | 19         | 9.31              | 19.61                  | $(3, 4]$                         | 3.49      |
| 5                       | 7          | 3.43              | 23.04                  | $(2.33, 3]$                      | 2.66      |
| 6                       | 26         | 12.75             | 35.78                  | $(1.86, 2.33]$                   | 2.10      |
| 7                       | 20         | 9.80              | 45.59                  | $(1.5, 1.86]$                    | 1.68      |
| 8                       | 25         | 12.25             | 57.84                  | $(1.22, 1.5]$                    | 1.36      |
| 9                       | 26         | 12.75             | 70.59                  | $(1, 1.22]$                      | 1.11      |
| 10                      | 42         | 20.59             | 91.18                  | $(0.82, 1]$                      | 0.91      |
| 11                      | 6          | 2.94              | 94.12                  | $(0.67, 0.82]$                   | 0.75      |
| 12                      | 4          | 1.96              | 96.08                  | $(0.54, 0.67]$                   | 0.61      |
| 13                      | 1          | 0.49              | 96.57                  | $(0.43, 0.54]$                   | 0.49      |
| 14                      | 7          | 3.43              | 100.00                 | $(0, 0.43]$                      | 0.22      |

Note: 'Cum. Percentage' represents the cumulative percentage.

Table 2.4.8 shows the mean and the median of the estimated loss aversion parameter  $\lambda$  in the two experiments. We can find that the corresponding statistics are very similar. Such a result is also verified by a Wilcoxon rank-sum test, which is used to test whether two independent groups are drawn from the same distribution. The p-value = 0.255 represents that we cannot reject the null hypothesis that the two distributions of loss aversion parameters from the 2014 and 2016 experiments are drawn from the same distribution. Figure 2.4.4 also graphically illustrates the two distributions.<sup>44</sup> The two distributions are both positively skewed.

<sup>44</sup> For the sake of consistency, we drop 5 extreme loss aversion coefficients ( $\lambda = 19.61$ ) while drawing the probability density distribution for the 2014 experiment.

**Result 5: Subjects have heterogeneous loss aversion parameters, but the distributions of the average loss aversion parameters are stochastically equivalent.**

Table 2.4.8 The description of loss aversion parameters in two experiments

| Experiment | # Subjects | Mean | S.D. | Median |
|------------|------------|------|------|--------|
| 2014       | 144        | 2.48 | 3.66 | 1.36   |
| 2016       | 60         | 2.31 | 2.10 | 1.68   |

*Note: S.D.: standard deviation.*

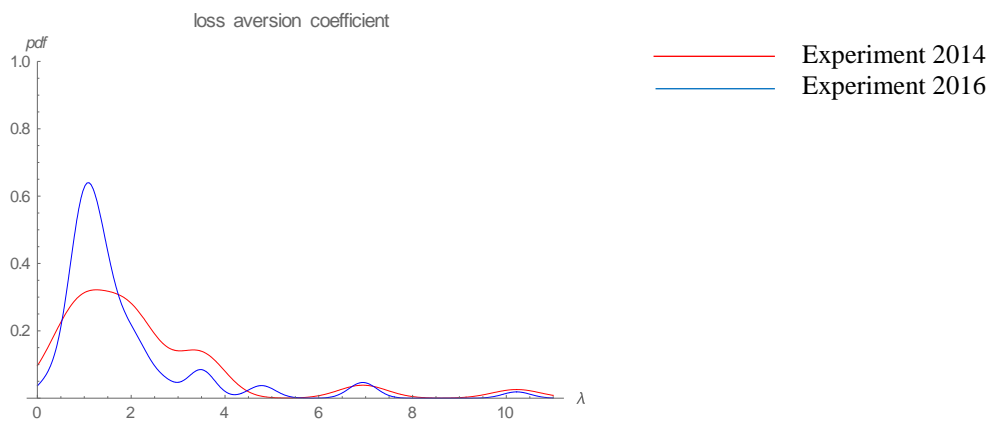


Figure 2.4.4 Probability density distributions for the estimated loss aversion parameters in the two experiments

## 2.5 Conclusion

In this chapter we re-examine a large amount of first-price independent private value auction experimental data published by CRS (1982), CSW (1983 & 1985). Treatment effect, which is a long time overlooked issue in the CRS (1982) experiment design has been identified. We find that in the 3-bidder market, the bidders bid less in this special first-price auction compared to the standard first-price auction; whereas such a difference is not significant in the 4-bidder market.

We also estimate heterogeneous constant relative risk aversion parameters under two assumptions:  $r_{max} = 1$  and  $r_{max} = 2$ . The results, on the one hand, confirm that there are heterogeneous risk aversion attitudes; on the other hand, also illustrate that the distribution of the risk attitudes is not influenced by whether we incorporate the possibility of risk-loving or not, and is also stable across the 3, 4, 5, 6, and 9 market sizes. This demonstrates the CRRA model is quite robust. In addition, we also find that the distributions of loss aversion coefficients are also quite similar across different experiments.

## **Chapter 3: Bidding behaviour in sequential first-price auctions with and without penalty: A re-examination**

### **3.1 Introduction**

In real-world auction markets, sellers sometimes need to sell two or more identical or non-identical objects. In multi-unit auctions, many options are open to the seller. First of all, the seller must decide whether to sell the objects jointly together in a single round or separately in a sequence of stages. Based on this difference, simultaneous auctions and sequential auctions are two general settings for multi-unit auctions. A dramatic difference between the two auction formats is the availability of information. In sequential auctions, normally the winning bid for each unit is revealed, whereas such information is unknown in simultaneous auctions.

The Shanghai license plate market, for instance, adopts the simultaneous auction format. At the same time, in another market in China - Kunming International Flower Auction market (KIFA), flowers are sold in sequential auctions with a Dutch auction format. Some other objects are also sold in a sequential auction setting such as - cattle, fish, vegetables, timber, tobacco, and wine. Besides this, in the financial market, sequential auctions are used by the government to privatise state-owned enterprises through selling the shares of Initial Public Offerings (IPO). Since sequential auctions are widely used in the field, we choose such an auction setting to study in this chapter.

Weber (1981) provides a theoretical benchmark model for sequential, independent private value auctions with single-unit demand risk-neutral bidders. He rigorously establishes that the sequence of prices in a sequential first-price or second-price auction is a ‘martingale’ - prices remain constant over time. Intuitively, on the one hand, the number of unsold items decreases with each completed auction round, which contributes to an

increasing price trend; on the other hand, fewer bidders still remain active in the market, which induces a decreasing price trend. These two effects offset each other. However, when relating the theoretical model to the field, a ‘declining price anomaly’<sup>45</sup> phenomenon, first described by Ashenfelter (1989), has been widely observed.<sup>46</sup> He studies the price patterns in a sequential English auction market for wine and finds that instead of the ‘law of one price’, actually the chance of the price decreasing is at least twice as great as the price increasing.

Ashenfelter (1989) informally proposes that risk aversion plays a significant role in real auction markets and may contribute to the anomaly. Nonetheless, McAfee and Vincent (1993) using a two-stage first-price and second-price auction model show that only by assuming the bidder displays a non-decreasing absolute risk aversion (NDARA), a symmetric increasing pure equilibrium bidding strategy could exist, and hence lead to the price declining. However, such an assumption is empirically implausible in reality since there appears to be a consensus that bidders display a decreasing absolute risk aversion.

In the same vein, Mezzetti (2011), and Hu and Zou (2015) both explore the ‘declining price anomaly’ from the view of risk attitudes. The former one assumes such an anomaly stems from an aversion to price risk; while the latter one formulates a general log-supermodular model to explain the anomaly. So far, there have been several other attempts to explain the ‘declining price anomaly’. For instance, Black and De Meza (1992) show through a two period second-price model that the price declines under a buyer’s

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<sup>45</sup> This is also known as the ‘afternoon effect’.

<sup>46</sup> Actually, the first empirical paper to investigate the price declining phenomenon is Buccola (1982), which reports such a finding in the livestock market. However, this paper does not use the term ‘declining price anomaly’ and has not drawn much attention in the literature.

option that allows the winner of the first item to buy the subsequent items at the same price.

Another strand of the literature that seeks to explain the phenomenon is supply uncertainty. Jeitschko (1999) considers a framework in sequential second-price auctions with two arrivals of information about supply during the course of a sequence. He finds that if the number of items for sale is larger than the number that bidders expect, prices would decline. Neugebauer and Pezanis-Christou (2007) introduce supply uncertainty in Weber's benchmark model for sequential first-price auctions; they show that the greater the uncertainty, the more the expected price declines. More recently, Rosato (2014) argued that reference-dependent preferences and loss aversion can rationalise the price declining phenomenon. He analyses two-stage first-price and second-price auction models, and shows that the higher the winning bid in the first stage, the less aggressive the bidding behaviour in the second stage. He concludes that this is because the bids are history-dependent and subject to a 'discouragement effect' in the second stage.

However, this chapter does not aim to find the reason behind the 'declining price anomaly'. Instead, we analyse an early stage laboratory experiment of sequential auctions conducted by Keser and Olson (1996).<sup>47</sup> Their paper contains important insights about the effect of bidders acting as agents in sequential auctions, which is inspired by Milgrom and Weber (1982) who suggest that the use of agents may result in declining prices. There are three treatments of sequential first-price auctions in their paper, which are benchmark, penalty, and agent treatment respectively. The benchmark treatment is a standard

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<sup>47</sup> Other experiments were conducted before Keser and Olson (1996). Burns (1985) considers a sequential English auction of professional wool buyers and students. Dorsey (1989) conducts a sequential auction in a laboratory setting after a single period first-price auction. Both of them find a declining price anomaly; however, in the experiment of Burns, the prices in the 'students' group eventually converge to a constant level.



sequential first-price auction. In both the penalty and the agent treatments, the bidders act as agents which is realised by imposing a penalty on the bidders who fail to acquire items. The only difference between these two treatments is that the penalty treatment allows bids greater than private values, whereas the agent treatment prohibits this. The design of the penalty is not just for reflecting a real-world auction situation. Moreover, it presents an implicit penalty borne by the bidders who lose the auctions.<sup>48</sup>

Therefore, the interest is on the bidders' behaviour when they face a monetary loss and how that would influence the seller's revenue. Previous experimental work has shown that losing and winning are generally not treated symmetrically by subjects. However, in the standard auction context, losing the auction does not involve any monetary loss. Therefore, in addition to the payback scheme analysed in Chapter 1, which involves subjects receiving an initial capital balance in a single-unit first-price auction and subsequently having to 'pay it back' if they lose, we also examine the penalties the losers must pay in the sequential first-price auction experiments conducted by Keser and Olson (1996). As a result, we gain a better understanding of how the bidders would behave in the auction setting with monetary loss and to check whether this accords with what the model predicts.

Besides presenting a risk neutral Nash equilibrium model without a strict argumentation, Keser and Olson (1996) also report some results of the penalty treatment. Firstly, they identify a declining price anomaly. Secondly, they compute an overall 98% allocation efficiency without taking the different auction stages into consideration. Thirdly, they estimate the bidding behaviour with respect to private value for each unit using linear and

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<sup>48</sup> By implicit penalty, we refer to the losing of market share when a buyer fails to purchase a unit (or a license).

quadratic functions and find that the slope coefficient is larger as the auction unravels which is as the model predicts.

The supply certainty treatment reported by Neugebauer and Pezanis-Christou (2007) is closely related to Keser and Olson (1996). It includes a longer auction series – 100 rounds instead of 20 rounds. In addition, Neugebauer and Pezanis-Christou (2007) also report two supply uncertainty treatments. As mentioned earlier, they extend the theoretical model to incorporate such a supply uncertainty. Since this chapter does not analyse the supply uncertainty treatment, we only summarise the findings of the certainty treatment in Neugebauer and Pezanis-Christou (2007).

With regard to prices, they find that the prices decline over the four stages for the first 20 rounds but remain constant for the 100 rounds overall. Unlike Keser and Olson (1996), they measure the allocation efficiency in terms of the percentage of occasions where the unit is sold to the bidder, whose value ranking is lower than or equal to the order of the unit, which yields a 72% aggregate efficiency. They also suggest that in 100-round auction, the high value bidders have time to learn the benefit of a ‘wait and see’ strategy – that is, not bidding aggressively in the early stages while giving the low value bidders a chance to win these early units. The ‘wait and see’ strategy can be observed from the relatively low allocation efficiencies for the first two units in the certainty treatment.

In this chapter, we adopt some methods used by Neugebauer and Pezanis-Christou (2007) to re-examine the experiments reported by Keser and Olson (1996). Our research question is whether a penalty would affect the auction market as predicted from the risk neutral Nash equilibrium (RNNE) model. To answer this question, it is necessary to separately look at five aspects: bidding strategy, price patterns, revenue, bid dependency, and allocation efficiency. Our main findings which are different from Keser and Olson (1996),

are summarised as follows: first of all, instead of estimating linear and quadratic bid functions, we estimate a piecewise linear regression assuming four equal segments and find that the bidding behaviour is non-monotonic, although the RNNE prediction for the intermediate value bidders' slope coefficient cannot be rejected.

Secondly, we identify that the price-declining anomaly stems from the price of the first unit being higher than the other three units. Thirdly, with the penalty, the seller's revenue indeed is greater than the no-penalty treatment as the model predicts. Fourthly, the bids depend not only on private values, but also on the price of the previous stage. Finally, by computing the allocation efficiency for each stage of the sequential auctions, we find a relative inefficiency for the first three units, which also suggests a 'wait and see' bidding strategy for the high value bidders. So overall, a penalty would affect the auction market differently from the RNNE prediction.

The remainder of the chapter is organised as follows. Section 3.2 introduces a penalty in the benchmark model and derives the theoretical framework. Section 3.3 describes the data. In Section 3.4, the analysis and results are reported. Section 3.5 concludes the chapter by recapping the important findings.

### 3.2 Model

Suppose there are  $n$  risk-neutral bidders with single-unit demand competing for  $k$  ( $k < n$ ) units of item. The auction lasts for  $k$  stages. In each stage, one unit is to be sold to the highest bidder, and the winning bid is publicly announced. All values are independently and identically drawn from a distribution  $F$  with density  $f$  defined on  $R_+$ . Each bidder has private information about his private value. The other parameters  $n$ ,  $k$  and  $F$  are common knowledge. An important feature of the scenario compared with the standard sequential auctions is that the bidders who have failed to purchase an item need to pay a penalty  $P$ .

Consider bidder 1 with private value  $x$ , and we define  $Y_1 > Y_2 \dots > Y_{n-1}$  as the order statistics of the remaining  $(n-1)$  other private values. Imagine we are at stage  $l$ , which implies that we have observed the price (winning bid) of stage  $1, 2, \dots, l-1$ . Under the assumption of a monotonic increasing bidding strategy in each auction stage, the bidders can infer the highest value of stage  $l-1$ ,  $y_{l-1}$ . In addition, the conditional distribution of the  $l$ th order statistic  $Y_l$  given the realization of  $Y_{l-1} = y_{l-1}$  and its density function are defined as

$$F_{Y_l}(z|y_{l-1}) = \left[ \frac{F(z)}{F(y_{l-1})} \right]^{n-l} \text{ and } f_{Y_l}(z|y_{l-1}) = \frac{(n-l)F(z)^{n-l-1}f(z)}{F(y_{l-1})^{n-l}}$$

Then we use backward induction to find out the symmetric bidding strategy in each auction stage, and so start from the last stage  $k$ . Suppose that all the bidders except for bidder 1 use the bidding strategy  $b_k(\cdot)$ . Bidder 1's expected profit from bidding  $b_k(\cdot)$  as if his value was  $z_k$  is given by the following expression

$$\Pi_k(z_k, x; y_1, y_2 \dots y_{k-1}) = \begin{cases} \int_0^{z_k} [x - b_k(z_k, y_1, y_2 \dots y_{k-1})] f_{Y_k}(\alpha | y_{k-1}) d\alpha \\ + \int_{z_k}^{y_{k-1}} (-P) f_{Y_k}(\alpha | y_{k-1}) d\alpha, & z_k \leq y_{k-1} \\ [x - b_k(z_k, y_1, y_2 \dots y_{k-1})], & z_k > y_{k-1} \end{cases}$$

The first equation represents the expected profit when bidder 1 bids according to a value  $z_k$  smaller than the highest value of stage  $-1$ ,  $y_{k-1}$ .  $P$  stands for the penalty the bidder faces if fails to acquire any item. The second equation stands for the expected profit when bidder 1 bids as if he had a value greater than  $y_{k-1}$  (which means greater than the values of all the remaining bidders). The first-order condition for profit maximization of truthful bidding requires  $b'_k(z_k, x; y_1, y_2 \dots y_{k-1})|_{z_k=x} = 0$ . For  $z_k \leq y_{k-1}$ , the first-order condition is

$$\begin{aligned} & -b'_k(x, y_1, y_2 \dots y_{k-1}) F(x)^{n-k} + [x - b_k(x, y_1, y_2 \dots y_{k-1})] (n-k) F(x)^{n-k-1} f(x) \\ & + P(n-k) F(x)^{n-k-1} f(x) = 0 \end{aligned}$$

yielding the following first-order differential equation below

$$b'_k(x, y_1, y_2 \dots y_{k-1}) = \begin{cases} [x - b_k(x, y_1, y_2 \dots y_{k-1}) + P] (n-k) \frac{f(x)}{F(x)}, & x \leq y_{k-1} \\ 0, & x > y_{k-1} \end{cases} \quad (3.2.1)$$

The boundary condition of the differential equation in equation (3.2.1) is  $b_k(0, y_1, y_2 \dots y_{k-1}) = 0$ . Furthermore, for any  $x \leq y_{k-1}$ , as the right-hand side of the equation is independent of  $y_1, y_2 \dots y_{k-1}$ , we have  $b_k(x, y_1, y_2 \dots y_{k-1}) = b_k(x)$ . For any  $x > y_{k-1}$ , we also have  $b_k(x, y_1, y_2 \dots y_{k-1}) = b_k(y_{k-1})$ . The last stage  $k$  equilibrium strategy is thus a function defined through  $b_k(\min\{x, y_{k-1}\})$ .

Next, we move to the second-to-last stage  $k - 1$ . Bidder 1 knows that if he loses in stage  $k - 1$  he will surely play the equilibrium strategy in stage  $k$ . In stage  $k - 1$ , bidder 1's expected profit takes the following expression

$$\Pi_{k-1}(z_{k-1}, x; y_1, y_2, \dots, y_{k-2}) = \begin{cases} \int_0^{z_{k-1}} [x - b_{k-1}(z_{k-1}, y_1, y_2, \dots, y_{k-2})] f_{Y_{k-1}}(\alpha | y_{k-2}) d\alpha \\ + \int_{z_{k-1}}^{y_{k-2}} [x - b_k(\min\{x, y_{k-1}\})] f_{Y_{k-1}}(\alpha | y_{k-2}) d\alpha, & z_{k-1} \leq y_{k-2} \\ [x - b_{k-1}(z_{k-1}, y_1, y_2, \dots, y_{k-2})], & z_{k-1} > y_{k-2} \end{cases}$$

If  $z_{k-1} \leq y_{k-2}$ , the first line stands for the expected profit if bidder 1 wins at stage  $k - 1$ , whereas the second line represents the expected profit if he loses at stage  $k - 1$  and he bids in equilibrium in stage  $k$ . On the other hand, by bidding  $z_{k-1} > y_{k-2}$ , he will certainly win. We note that the penalty  $P$  does not enter into the expected profit function for stage  $k - 1$ , which is because even if bidder 1 loses at this stage, he still could win in the last stage. Therefore, the penalty  $P$  should only exist in the expected profit for the last stage  $k$ . The first-order condition for maximization of the bidder's expected profit for stage  $k - 1$  also requires  $b'_{k-1}(z_{k-1}, y_1, y_2, \dots, y_{k-2})|_{z_{k-1}=x} = 0$ . For  $z_{k-1} \leq y_{k-2}$ , the first-order condition is

$$\begin{aligned} & -b'_{k-1}(x, y_1, y_2, \dots, y_{k-2}) F(x)^{n-k+1} + [x - b_{k-1}(x, y_1, y_2, \dots, y_{k-2})] (n - k + 1) F(x)^{n-k} f(x) \\ & - [x - b_k(\min\{x, y_{k-1}\})] (n - k + 1) F(x)^{n-k} f(x) = 0 \end{aligned}$$

yielding the following first-order differential equation below

$$b'_{k-1}(x, y_1, y_2, \dots, y_{k-1}) = \begin{cases} [b_k(x) - b_{k-1}(x, y_1, y_2, \dots, y_{k-2})](n-k+1) \frac{f(x)}{F(x)}, & x \leq y_{k-2} \\ 0, & x > y_{k-2} \end{cases} \quad (3.2.2)$$

As in stage  $k$ , the equilibrium bidding strategy for stage  $k-1$  is independent of  $y_1, y_2, \dots, y_{k-2}$  for any  $x \leq y_{k-2}$ ; therefore we have  $b_{k-1}(x, y_1, y_2, \dots, y_{k-2}) = b_{k-1}(x)$ . For any  $x > y_{k-2}$ , we also have  $b_{k-1}(x, y_1, y_2, \dots, y_{k-2}) = b_{k-1}(y_{k-2})$ . The second-to-last stage  $k-1$  equilibrium strategy is thus a function defined through  $b_{k-1}(\min\{x, y_{k-2}\})$ .

The equilibrium bidding strategies for the stage  $k-2$ , as well as the previous stages are determined in a similar way. To summarise, in each stage  $l$ , the bidding strategy is  $b_l(\min\{x, y_{l-1}\})$ , for  $l = 1, 2, \dots, k$ . Here, it is noted that for the first stage strategy,  $y_0$  actually is the upper bound of the private values that the bidders do not need to infer. Besides this, the bidding strategy is monotonic increasing in private value. For all  $l$ , it should always have  $x \leq y_{l-1}$ . Therefore,  $b_l(\min\{x, y_{l-1}\}) = b_l(x)$ .

Having derived the risk neutral Nash equilibrium bidding strategy in each stage of the sequential auctions with penalty, we implement this rule to pin down the bidding strategy in the experiment conducted by Keser and Olson (1996).<sup>49</sup> In this experiment, the private values are i.i.d drawn from  $[1, 1000]$ .

$$F(x) = \frac{x-1}{999}, f(x) = \frac{1}{999}$$

Solving equation (3.2.1) for stage  $k$  yields

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<sup>49</sup> Keser and Olson (1996) show the RNNE bidding strategy but not the derivation process. They state that the penalty “simply shifts the risk neutral bidding functions upward by the amount of the penalty.”

$$b_k'(x)F(x) = [x - b_k(x) + P](n-k)f(x)$$

$$b_k'(x)F(x) + b_k(x)(n-k)f(x) = x(n-k)f(x) + P(n-k)f(x)$$

Multiplying both sides by  $F(x)^{n-k-1}$ , we obtain

$$b_k'(x)F(x)^{n-k} + b_k(x)(n-k)F(x)^{n-k-1}f(x) = x(n-k)F(x)^{n-k-1}f(x) + P(n-k)F(x)^{n-k-1}f(x)$$

$$\frac{\partial}{\partial x}(F(x)^{n-k}b_k(x)) = x(n-k)F(x)^{n-k-1}f(x) + P(n-k)F(x)^{n-k-1}f(x)$$

Integrating from 1 to  $x$ , we get

$$b_k(x) = \frac{n-k}{F(x)^{n-k}} \left[ \int_1^x t F(t)^{n-k-1} f(t) dt + \int_1^x P F(t)^{n-k-1} f(t) dt \right]$$

$$= \frac{(n-k)999^{n-k}}{(x-1)^{n-k}} \left[ \int_1^x \frac{(t-1)^{n-k-1}}{999^{n-k-1}} \frac{1}{999} dt + \int_1^x P \frac{(t-1)^{n-k-1}}{999^{n-k-1}} \frac{1}{999} dt \right]$$

$$= \frac{(n-k)(999^{n-k})}{(999^{n-k})(x-1)^{n-k}} \left[ \int_1^x t(t-1)^{n-k-1} dt + P \int_1^x (t-1)^{n-k-1} dt \right]$$

$$= \frac{(n-k)}{(x-1)^{n-k}} \left[ \frac{(x-1)^{n-k+1}}{n-k+1} + \frac{(x-1)^{n-k}}{n-k} + P \frac{(x-1)^{n-k}}{n-k} \right]$$

$$b_k(x) = P + \frac{n-k}{n-k+1}(x-1) + 1$$

By moving backwards to stage  $k-1$  to solve equation (3.2.2), we get

$$b_{k-1}'(x)F(x) + b_{k-1}(x)(n-k+1)f(x) = b_k(x)(n-k+1)f(x)$$

Multiplying both sides by  $F(x)^{n-k}$ , the equation becomes



$$\begin{aligned}
b_{k-1}'(x)F(x)^{n-k+1} + b_{k-1}(x)(n-k+1)F(x)^{n-k}f(x) &= b_k(x)(n-k+1)F(x)^{n-k}f(x) \\
\frac{\partial}{\partial x}(F(x)^{n-k+1}b_{k-1}(x)) &= b_k(x)(n-k+1)F(x)^{n-k}f(x) \\
b_{k-1}(x) &= \frac{n-k+1}{F(x)^{n-k+1}} \int_1^x F(t)^{n-k} b_k(t) f(t) dt \\
&= \frac{(n-k+1)(999^{n-k+1})}{(x-1)^{n-k+1}} \int_1^x \frac{(t-1)^{n-k}}{999^{n-k+1}} \left( P + \frac{n-k}{n-k+1}(t-1) + 1 \right) dt \\
&= \frac{n-k+1}{(x-1)^{n-k+1}} \left[ P \int_1^x (t-1)^{n-k} dt + \frac{n-k}{n-k+1} \int_1^x (t-1)^{n-k+1} dt + \int_1^x (t-1)^{n-k} dt \right] \\
&= \frac{n-k+1}{(x-1)^{n-k+1}} \cdot P \cdot \frac{(x-1)^{n-k+1}}{n-k+1} + \frac{n-k+1}{(x-1)^{n-k+1}} \cdot \frac{n-k}{n-k+1} \cdot \frac{(x-1)^{n-k+2}}{n-k+2} + \\
&\quad \frac{n-k+1}{(x-1)^{n-k+1}} \cdot \frac{(x-1)^{n-k+1}}{n-k+1} \\
b_{k-1}(x) &= P + \frac{n-k}{n-(k-1)+1}(x-1) + 1
\end{aligned}$$

**Proposition.** Let  $k$  be the total number of items to be sold, and  $P$  the penalty that needs to be paid by the bidder who fails to obtain an item. For sequential first-price auctions with unit-demand and private value bidders, there exists a series  $(b_1, b_2, \dots, b_k)$  of symmetric monotonically increasing equilibrium bidding strategies defined as

$$\begin{aligned}
b_{k-l}(x) &= P + \frac{n-k}{n-(k-l)+1}(x-1) + 1, \\
&\text{for } l = 0, 1, 2, \dots, k-1
\end{aligned} \tag{3.2.3}$$

In equilibrium, the expected price for each stage  $k-l$ ,  $l = 0, 1, 2, \dots, k-1$  is determined by evaluating the corresponding Nash equilibrium bidding strategy at the expected highest value in the uniform distribution  $[1, 1000]$  in the given stage, which is  $\frac{n-(k-l-1)}{n+1} \times 999$ , for  $l = 0, 1, 2, \dots, k-1$ . Hence, the seller's expected price for each stage  $k-l$ ,  $l = 0, 1, 2, \dots, k-1$  is as follows

$$\begin{aligned}
p_{k-l} &= P + \frac{n-k}{n-(k-l)+1} \times \left[ \frac{n-(k-l-1)}{n+1} \times 999 - 1 \right] \\
&= P + \frac{n-k}{n+1} \times 999 + \frac{l+1}{n-k+l+1}, \\
&\text{for } l = 0, 1, 2, \dots, k-1
\end{aligned} \tag{3.2.4}$$

Therefore, to compute the revenue in sequential auctions, we only need to sum the prices for all four stages, which is given by:

$$R = Pk + \frac{(n-k)k}{n+1} \times 999 + \frac{(l+1)k}{n-k+l+1} \tag{3.2.5}$$

In Keser and Olson's (1996) experiment,  $n = 8$ ,  $k = 4$ , and  $P = 100$ . We substitute the corresponding parameters in equation (3.2.3) – (3.2.5), and then summarise the predicted results in Table 3.2.1.

Table 3.2.1 RNNE predicted bidding strategy and price for each stage and revenues for both treatments

| Treatment |   | Penalty                  |       |        | No-Penalty             |       |        |
|-----------|---|--------------------------|-------|--------|------------------------|-------|--------|
|           |   | $b(x)^*$                 | $p^*$ | $R^*$  | $b(x)^*$               | $p^*$ | $R^*$  |
| Stage     | 1 | $\frac{1}{2}(v-1) + 101$ | 544.5 |        | $\frac{1}{2}(v-1) + 1$ | 444.5 |        |
|           | 2 | $\frac{4}{7}(v-1) + 101$ | 544.4 |        | $\frac{4}{7}(v-1) + 1$ | 444.4 |        |
|           |   |                          |       | 2177.4 |                        |       | 1777.4 |
|           | 3 | $\frac{2}{3}(v-1) + 101$ | 544.3 |        | $\frac{2}{3}(v-1) + 1$ | 444.3 |        |
|           | 4 | $\frac{4}{5}(v-1) + 101$ | 544.2 |        | $\frac{4}{5}(v-1) + 1$ | 444.2 |        |

From Table 3.2.1, with a penalty the bidders are worse off since for each stage, as their bids increase by exactly as much as the penalty  $P=100$ . At the same time, the auction seller is better off since he gets an extra 400 from the four winners in the sequential auctions.

### 3.3 Data description

The data used in this chapter is from Keser and Olson (1996). The aim of their research is to conduct a series of experiments to analyse the declining price phenomenon in sequential auctions. They conduct two novel treatments where the bidders act as agents, thus allowing them to check if that is a possible contributing factor to the declining price phenomenon. More specifically, there are three treatments in their experiment, and they use between-subject design, which means one subject can only participate in one treatment. The three treatments are baseline no-penalty treatment (NP treatment), penalty treatment (P treatment) and agent treatment (AP treatment), respectively. For the latter two treatments, the bidders both act as agents. The only difference is that the penalty treatment allows bids greater than the private values, whereas the agent treatment prohibits this.

To create an environment where the bidders can act as agents, they impose a penalty for the bidder to pay whenever he fails to acquire an item. In this chapter, we re-examine the data provided by Keser and Olson (1996) because we are interested in one research question - when bidders face a penalty, would they bid as the RNNE model predicts? We have analysed the influence of a payback scheme in the first-price private value auctions in Chapter 1. The main similarity of these two experimental designs is that the losers both need to pay some money. However, the difference is that in the payback scheme

experiment, the losers need to pay the initial capital balance back to the auction seller, whereas in the penalty treatment, the losers need to pay the penalty to the ‘principal’. Therefore, by combining and contrasting the findings from Chapter 1 with our analysis of Keser and Olson’s P treatment, an improved understanding of how bidders respond to a monetary loss in the auction context can be achieved.

We derive the RNNE bidding strategy for the NP and P treatments in Section 3.2 above. However, it is difficult to pin down the Nash equilibrium bidding strategy for the AP treatment with the confounding effects of the penalty as well as a restriction on the bids.

The no-penalty treatment (NP treatment) is composed of four sessions. In each session, eight single-unit demand bidders compete for four identical items. The selling of the four units is through a format of sequential auction which includes four stages, and in each stage one unit is sold. The eight bidders’ private values are drawn from  $[1, 1000]$  tokens with an identical chance of any integer. At the beginning of the auction, each bidder knows his private value for the item. In the first stage, each bidder submits a bid for the first unit.

The winner is the one who submits the highest bid, and his bid is disclosed before the start of the second stage. The winner acquires the first item at the price he bids and does not participate in the rest of the auction. In the second stage, each remaining bidder submits a bid for the second unit, and the bidder with the highest bid wins the unit with his bid being revealed to the others. At the same time, his participation in the auction is over. The same process continues for the third and the last auction stage, after which the auction concludes. Such a sequential auction is repetitively conducted for twenty rounds, and before each auction, a private value is drawn from the same distribution  $[1, 1000]$  tokens for each bidder.

The penalty treatment (P treatment) is similar to the NP treatment, except that the four losers who fail to obtain a unit need to pay a penalty worth 100 tokens each. Both treatments allow the bidder to submit bids greater than their private values.

### 3.4 Analysis

In this section, the main research question is would the existence of a penalty influence the bidders' behaviour as predicted in Section 3.2? To address this issue, we discuss five conjectures in each subsequent section. The conjectures for the P treatment are as follows:

- Conjecture 1: The bidders use a monotonic increasing bidding strategy.
- Conjecture 2: The price is constant for all four stages.
- Conjecture 3: The seller's revenue is greater compared with the NP treatment.
- Conjecture 4: The bids are independent from the price of the previous stage.
- Conjecture 5: The allocation efficiency is 100% for all four stages.

Keser and Olson (1996) also discuss Conjectures 2 and 5 in their paper. More specifically, they identify that the average price decrease for the four stages as well as the first two units' prices are not significantly different by using a parametric test. With regards to the allocation efficiency, they treat a sequential auction as efficient as long as the four winners are the bidders with the four highest values, while not taking the order of the four units into consideration. Besides the discussion of these two conjectures, they also estimate the linear and quadratic bidding functions, which is relevant to Conjecture 1 in this chapter. However, they do not check the monotonicity. Different from their methods, we will explore the answers to such questions drawing on the techniques adopted by Neugebauer and Pezanis-Christou (2007), such as piecewise linear bidding estimation, nonparametric

tests for independent or related samples (see Siegel & Castellan, 1988), and treating each auction stage separately when measuring the allocation efficiency.<sup>50</sup>

### **3.4.1 Piecewise bid functions**

For the auction of each unit in the P treatment, if the bidders bid according to the risk neutral Nash Equilibrium bidding strategy, bidding functions would be monotonic increasing in values. Then the bidding curve should be linear with respect to each auction stage. The bidder with the highest private value wins the first unit, and the second highest private value bidder wins the second unit, and so on. Therefore, the allocation efficiency should be 100%.

Keser and Olson (1996) use OLS regression to estimate both linear and quadratic bid functions without an intercept, and find that the quadratic bid function fits the data better. At the same time, they identify that the bidders increase their bids for each subsequent unit as predicted. In addition to their analysis, we estimate a piecewise linear bidding function to check whether bidders use a monotonic increasing bid strategy. Here we adopt the same method as Neugebauer and Pezanis-Christou (2007) who assume four equal segments for their piecewise linear bidding function estimation. The four segments of our bidding function are [1,250] as the low value range; (250,500] and (500,750] as the first and the second half of the intermediate value range; and (750,1000] as the high value range. The estimated piecewise linear bidding functions are reported in Table 3.4.1. We first analyse the P treatment since the bidding behaviour in this treatment is our main research interest. Note that even though there is no theoretical prediction for the AP treatment, it is still beneficial to consider the corresponding results together with the P

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<sup>50</sup> The nonparametric techniques do not assume any distribution and only rely on ranking tests, and are useful with small samples.

treatment. Secondly, we consider the NP treatment and compare it with a similar design by Neugebauer and Pezanis-Christou (2007).

*Intercept:*

- For the P treatment, the estimated intercepts for all four units are much less than the RNNE prediction (where penalty  $P = 100$ ), which vary from 25.6% to 53.8% of the penalty. This result is quite similar to what we identify in Chapter 1 of the first-price auction with the payback scheme. In that experiment, the intercepts of the bidding functions are also smaller than the prediction. This common finding implies that when the subjects face a monetary loss, they would not change their bids as much as predicted.
- For the AP treatment, the estimated intercepts for most units are insignificant except for the second unit, which is significantly negative.

*Slope:*

- For the P treatment, while the slope coefficients for most segments and units are positive, they are insignificant for the last segment over all four units. This violates our assumption of a monotonic increasing bidding strategy. With regards to the comparison between the bid slope and the RNNE prediction for each stage as shown in Table 3.2.1, we have
  - $Bid > RNNE$ , for the low value bidders
  - $Bid = RNNE$ , for the intermediate value bidders<sup>51</sup>
  - $Bid < RNNE$ , for the high value bidders.

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<sup>51</sup> The estimated slope for last unit bids of the bidders within the third segment is an exception as it is smaller than the RNNE prediction.

- For the AP treatment, the slope coefficients for the last segment are also insignificant, except that the last unit is significantly positive.

**Result (non-monotonicity in the P & AP treatments):** The bidders use a non-monotonic increasing bid strategy, although this doesn't hold for the last unit in the AP treatment.

Figure 3.4.1 and 3.4.2 plot the estimated bidding functions for the P and AP treatment, respectively. Each graph plots actual bids against private values for a given unit. The green 'RNNE' line represents the RNNE-predicted bidding function. The red 'B=PV' line represents all points where bid equals private value. The black 'Est.' line represents the estimated piecewise linear bidding function.

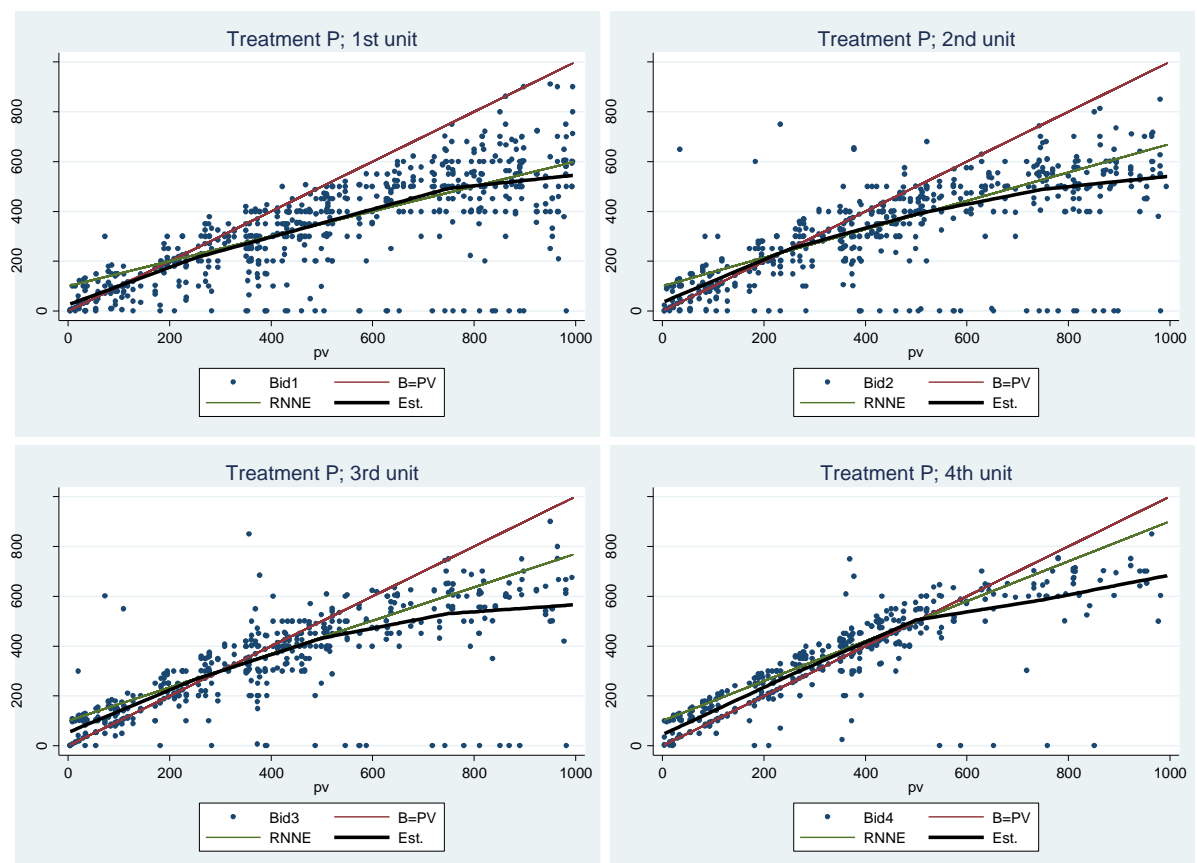


Figure 3.4.1 Estimated piecewise linear bid functions, bids, and RNNE predictions in the P treatment



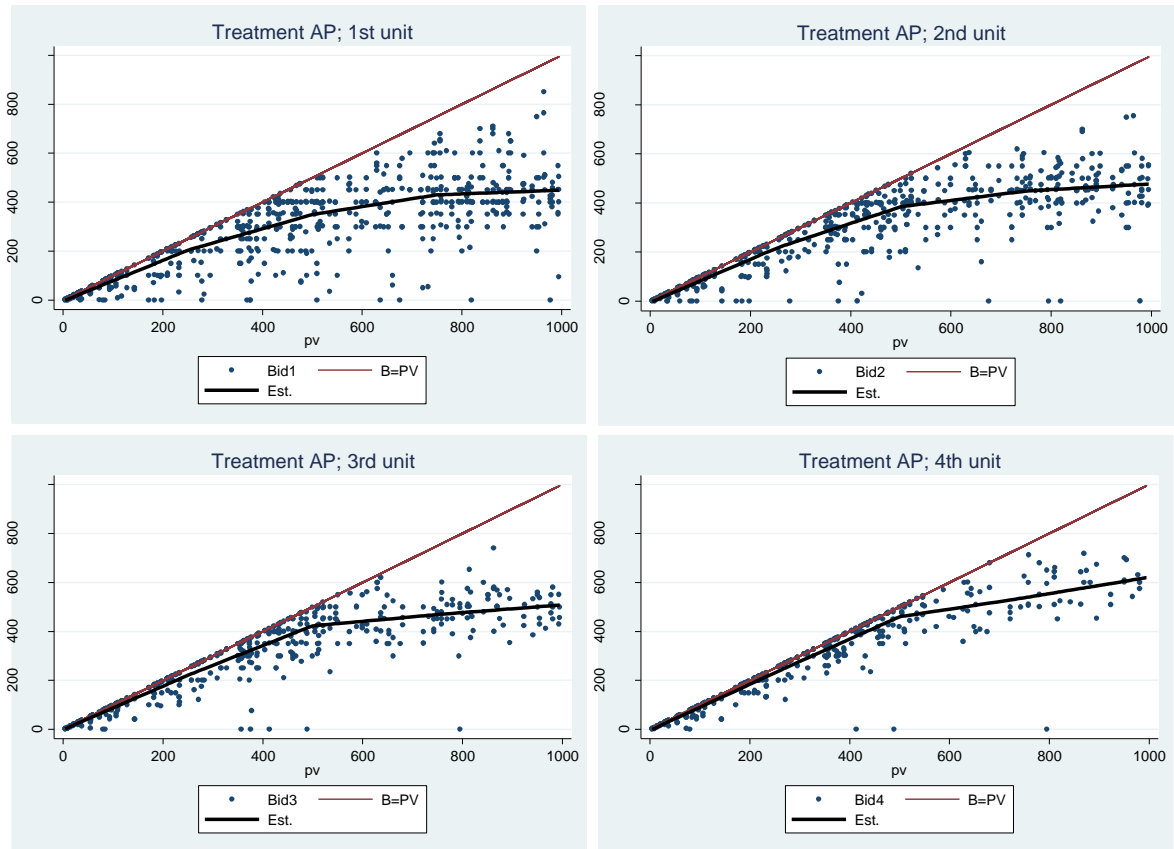


Figure 3.4.2 Estimated piecewise linear bid functions and bids in the AP treatment

Having analysed the two treatments with penalty, it is necessary to identify whether the bids in the NP treatment follow a monotonic increasing bid strategy. From Table 3.4.1, it can be seen that this is the case since the slope coefficients are positive.<sup>52</sup>

**Result (monotonicity in the NP treatment):** The bidders use a monotonic increasing bid strategy.

Such a finding is different from the experiment result reported by Neugebauer and Pezanis-Christou (2007). In their supply certainty treatment, the bidders use a non-

<sup>52</sup> The slope coefficient for the 3rd unit in the range of (500,750] is insignificant. However, following the same method as Neugebauer and Pezanis-Christou (2007) who deal with this issue, we can still say the NP treatment bids are monotonic increasing.

monotonic bidding strategy not only over all 100 rounds but also more specifically for the last 25 rounds.<sup>53</sup>

***Result (bid slope vs prediction in the NP treatment):***

- Bid > RNNE, for the low value and the first half of the intermediate value bidders
- Bid < RNNE, for the second half of the intermediate value and the high value bidders

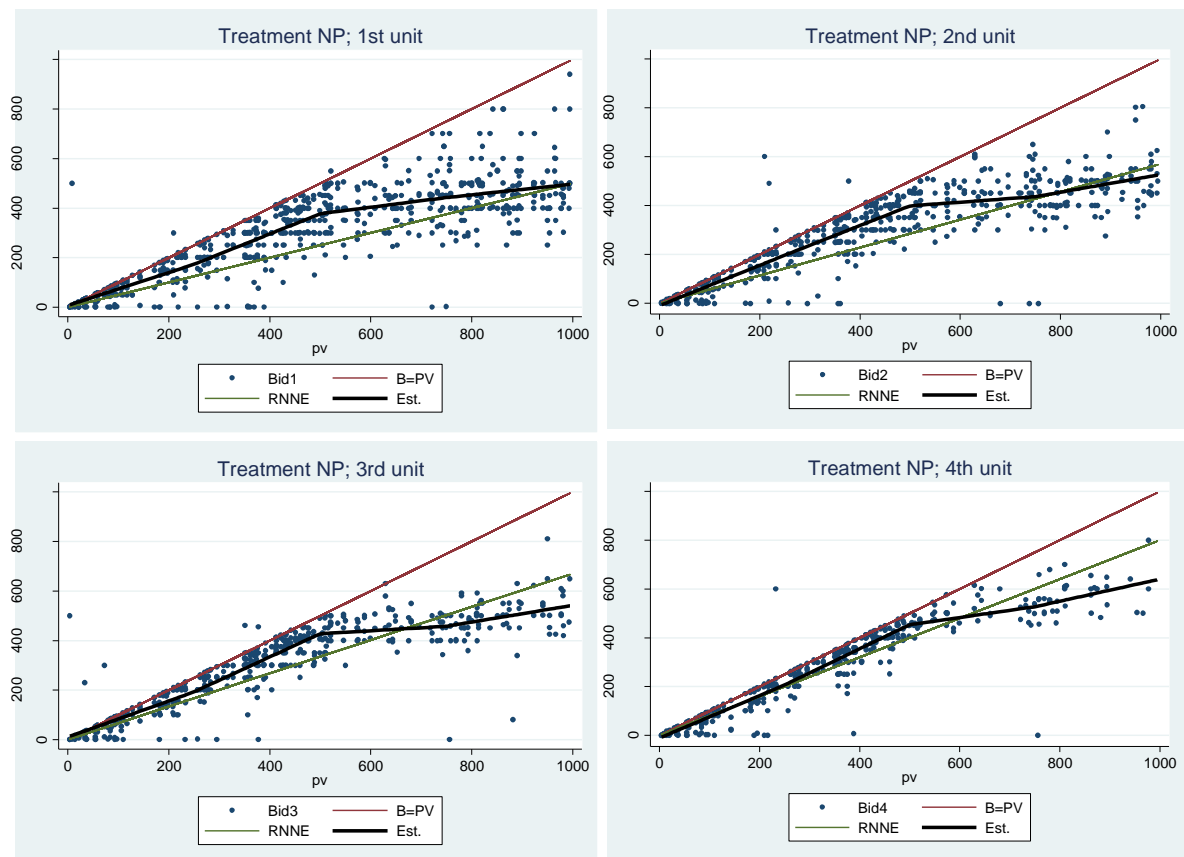


Figure 3.4.3 Estimated piecewise linear bid functions, bids, and RNNE predictions in the NP treatment

<sup>53</sup> The slope coefficients of all units are insignificant for the last segment private values.

Table 3.4.1 Estimated piecewise linear functions

| Treat<br>ment<br>Unit  | P                               |                                |                                 |                                 | AP                                |                                 |                                  |                                  | NP                               |                                   |                                    |                             |
|------------------------|---------------------------------|--------------------------------|---------------------------------|---------------------------------|-----------------------------------|---------------------------------|----------------------------------|----------------------------------|----------------------------------|-----------------------------------|------------------------------------|-----------------------------|
|                        | 1st                             | 2 <sup>nd</sup>                | 3rd                             | 4 <sup>th</sup>                 | 1st                               | 2nd                             | 3rd                              | 4th                              | 1st                              | 2nd                               | 3rd                                | 4th                         |
| Int.                   | 25.61<br>[14.31,<br>36.91]      | 35.29<br>[11.72,<br>58.85]     | 53.80<br>[32.33,<br>75.28]      | 45.01<br>[30.58,<br>59.44]      | <b>-4.25</b><br>[-10.67,<br>2.18] | -5.61<br>[-10.58,<br>-0.63]     | <b>-1.23</b><br>[-5.24,<br>2.79] | <b>-3.71</b><br>[-7.50,<br>0.08] | <b>7.78</b><br>[-9.13,<br>24.68] | <b>-8.66</b><br>[-17.35,<br>0.03] | <b>11.01</b><br>[-16.21,<br>38.23] | -10.56<br>[-16.4,<br>-4.71] |
| 1                      | 0.75<br>[0.66,<br>0.84]         | 0.86<br>[0.71,<br>1.01]        | 0.85<br>[0.74,<br>0.96]         | 0.95<br>[0.85,<br>1.05]         | 0.83<br>[0.74,<br>0.92]           | 0.88<br>[0.82,<br>0.94]         | 0.89<br>[0.83,<br>0.95]          | 0.94<br>[0.90,<br>0.98]          | 0.66<br>[0.52,<br>0.81]          | 0.82<br>[0.69,<br>0.96]           | 0.73<br>[0.57,<br>0.89]            | 0.87<br>[0.77,<br>0.97]     |
| 2                      | 0.56<br>[0.42,<br>0.70]         | 0.56<br>[0.34,<br>0.78]        | 0.66<br>[0.49,<br>0.83]         | 0.89<br>[0.73,<br>1.05]         | 0.58<br>[0.44,<br>0.73]           | 0.68<br>[0.59,<br>0.77]         | 0.81<br>[0.72,<br>0.89]          | 0.91<br>[0.82,<br>1.01]          | 0.82<br>[0.71,<br>0.93]          | 0.81<br>[0.67,<br>0.95]           | 0.94<br>[0.85,<br>1.03]            | 0.99<br>[0.86,<br>1.11]     |
| 3                      | 0.56<br>[0.36,<br>0.76]         | 0.40<br>[0.11,<br>0.69]        | 0.40<br>[0.08,<br>0.71]         | <b>0.32</b><br>[-0.05,<br>0.70] | 0.32<br>[0.18,<br>0.47]           | 0.26<br>[0.14,<br>0.38]         | 0.18<br>[0.03,<br>0.34]          | 0.31<br>[0.04,<br>0.58]          | 0.26<br>[0.12,<br>0.40]          | 0.15<br>[0.03,<br>0.27]           | <b>0.12</b><br>[-0.01,<br>0.25]    | 0.29<br>[0.07,<br>0.52]     |
| 4                      | <b>0.21</b><br>[-0.02,<br>0.45] | <b>0.21</b><br>[0.00,<br>0.42] | <b>0.14</b><br>[-0.10,<br>0.39] | <b>0.40</b><br>[-0.16,<br>0.95] | <b>0.08</b><br>[-0.15,<br>0.31]   | <b>0.11</b><br>[-0.05,<br>0.28] | <b>0.16</b><br>[-0.01,<br>0.32]  | 0.34<br>[0.06,<br>0.62]          | 0.22<br>[0.03,<br>0.41]          | 0.36<br>[0.17,<br>0.56]           | 0.34<br>[0.10,<br>0.57]            | 0.45<br>[0.00,<br>0.9]      |
| # Obs                  | 640                             | 560                            | 480                             | 400                             | 640                               | 560                             | 480                              | 400                              | 640                              | 560                               | 480                                | 400                         |
| Adj.<br>R <sup>2</sup> | 0.60                            | 0.55                           | 0.60                            | 0.77                            | 0.64                              | 0.75                            | 0.84                             | 0.90                             | 0.72                             | 0.79                              | 0.85                               | 0.90                        |

Note: 'Int.' means intercept; '1', '2', '3', and '4' are the estimated slopes for the four segments [1,250], (250,500], (500,750], and (750,1000] respectively. Bold figures indicate that the estimates are not significantly different from 0 at  $\alpha = 0.05$ . The numbers in square brackets indicate the 95% confidence interval for each estimate.

This sharp difference in the slopes of the estimated bid functions for private values greater or smaller than 500 is also visually shown in Figure 3.4.3. The major difference between these slopes is also identified by Neugebauer and Pezanis-Christou (2007) in their experiment, which naturally formed the basis for separately analysing high and low value bidders' behaviour in their paper. They use the word "wait-and-see" to describe the behaviour for the high value bidders who do not bid as aggressively as low value bidders.

*Intercept:* The intercepts for the first three units are not significantly different from zero and this is correctly predicted by RNNE, whereas it is significantly negative for the last unit. Neugebauer and Pezanis-Christou (2007) report a different result, which is for most units, the intercepts are significantly negative.

To address the question of how adding the penalty  $P$  in the sequential auction would influence the bidders' behaviour in the  $P$  treatment, we conclude as follows:

- The bidders use non-monotonic increasing bid strategies in all four stages.
- The intercepts of the bidding strategies across all four stages are all much smaller than the value of  $P$ .

Therefore, it can be said that the penalty does not influence the bids as we predicted from the theoretical framework.

### **3.4.2 Price behaviour**

There are two questions that need to be answered in this section:

- Are the prices of four units' constant?
- In each auction stage, is the price the same as the RNNE prediction?

From Table 3.4.2, we can see that in all three treatments, the mean prices over the four units are decreasing.<sup>54</sup> Keser and Olson (1996) have also identified the declining price phenomenon. They find that decreasing prices are more common from unit 1 to 2 and unit 2 to 3, and less common from unit 3 to 4 in all three treatments.

Table 3.4.2 Observed average prices in the three treatments

| Treatment       | P                        | AP    | NP    | P-NP                    | P-AP  | AP-NP                    |
|-----------------|--------------------------|-------|-------|-------------------------|-------|--------------------------|
| Unit            |                          |       |       |                         |       |                          |
| 1 <sup>st</sup> | 615.1                    | 510.4 | 544.0 | 71.1                    | 104.7 | <b>-33.6<sup>b</sup></b> |
| 2 <sup>nd</sup> | 570.3                    | 492.8 | 504.1 | 66.2                    | 77.5  | <b>-11.3<sup>b</sup></b> |
| 3 <sup>rd</sup> | <b>561.9<sup>a</sup></b> | 476.1 | 482.9 | <b>79.0<sup>a</sup></b> | 85.8  | <b>-6.8<sup>b</sup></b>  |
| 4 <sup>th</sup> | <b>559.4<sup>a</sup></b> | 476.0 | 472.1 | <b>87.3<sup>a</sup></b> | 83.4  | <b>3.9<sup>b</sup></b>   |

*Note: The observed average price for each unit in the corresponding treatment is the realized average price across all four sessions. Bold figures with 'a' indicate that the estimates are not significantly different from the RNNE predictions at  $\alpha = 0.01$ . Bold figures with 'b' indicate that the estimates are not significantly different from zero at  $\alpha = 0.01$ .*

However, it is necessary to use a nonparametric Friedman test to check whether statistically the prices for all four units are drawn from the same population or not. From the results of this test, we cannot reject that in the AP treatment, the prices are constant, whereas for both the P and NP treatments, the null hypothesis is rejected at the 1% level, which means at least one unit price is not from the same population as the other unit prices. Dunn's test has been used to pin down which unit price is from a different population.<sup>55</sup> The result shows that the first unit price is significantly different from the other three units at the 5% level whereas the prices of the other three units are from the same population.

<sup>54</sup> In both NP and AP treatments, the observed average prices of all four units reported in Table 3 of Keser and Olson (1996, p. 165) are different from our results.

<sup>55</sup> Dunn's test is a nonparametric test used when Kruskal–Wallis test or Friedman's test has been rejected, see Dinno (2015).

This shows that in both the P and NP treatments, the price declining phenomenon actually only significantly exists for the first two stages.

**Result AP treatment (constant price):**  $P_{AP}^1 = P_{AP}^2 = P_{AP}^3 = P_{AP}^4$

**Result P & NP treatments (declining price trend):**  $P_P^1 > P_P^2 = P_P^3 = P_P^4$

$$P_{NP}^1 > P_{NP}^2 = P_{NP}^3 = P_{NP}^4$$

After addressing the price trend, we check if each unit's realised price is the same as predicted in Table 3.2.1. We use the Wilcoxon signed rank test and find that the prices of the first two units are significantly higher than the RNNE predictions for both the P and NP treatments. However, for the last two unit's different results were obtained. That is, the prices of the last two units are significantly higher than predicted in the NP treatment, but not in the P treatment.

**Result (price vs prediction in the P treatment):**

$$P_P^1 > P_P^{1*}$$

$$P_P^2 > P_P^{2*}$$

$$P_P^3 = P_P^{3*}$$

$$P_P^4 = P_P^{4*}$$

**Result (price vs prediction in the NP treatment):**

$$P_{NP}^1 > P_{NP}^{1*}$$

$$P_{NP}^2 > P_{NP}^{2*}$$

$$P_{NP}^3 > P_{NP}^{3*}$$

$$P_{NP}^4 > P_{NP}^{4*}$$

It is instructive to compare the result of the NP treatment with Neugebauer and Pezanis-Christou's (2007) supply certainty treatment. In Neugebauer and Pezanis-Christou's

(2007) experiment, the first 20 rounds also shows a declining price trend as in the NP treatment. However, when it comes to the 100 rounds overall, they identify a trend-free price. As they summarise, ‘With experience, behaviour settled into a heuristic that leads to the law of one price.’ Besides the price trend, they also find that the prices of the four units are significantly higher than predicted which accords with the NP treatment.

It is also useful to conduct a cross-treatment test. Since we do not have theoretical predictions for the AP treatment, we intuitively compare the realized price for each unit in this treatment with the corresponding results in the P and NP treatments. As a result, using the ranksum test, the prices in the AP treatment are always significantly smaller than the P treatment, but are not significantly different from the NP treatment.

**Result (across AP and P treatments):**  $P_{AP} < P_P$

**Result (across AP and NP treatments):**  $P_{AP} = P_{NP}$

For the P and NP treatments, not surprisingly, the prices of all four units in the P treatment are higher than in the NP treatment at a 1% level of significance. We also want to quantitatively pin down the actual difference and to see if that is the same as we predicted from the RNNE model, which is 100. Comparing the price difference of each unit across the two treatments, it is significantly smaller than predicted for the first two units whereas it is not significantly different from our prediction for the last two units.<sup>56</sup>

**Result (across P and NP treatments):** The price differences for the first two units are smaller than the RNNE prediction whereas the price differences for the last two units are not significantly different from the prediction.

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<sup>56</sup> We get the same result by running a regression.

### 3.4.3 Seller's revenue

In this section, we restrict our analysis to two treatments only - NP and P, and look at how imposing a penalty would influence the seller's revenue and whether the influence is the same as the RNNE model predicts. Therefore, in conjunction with Chapter 1, which analyses the impact of a payback scheme given to the bidders in first-price private value auctions, we have a full picture of how a device leading to the monetary loss to the bidders would affect the seller's revenue.

We use a pooled OLS regression model which includes a dummy variable for the treatment to test the following two hypotheses

$$H1_0 : R_{NP} = 1777.4$$

$$H2_0 : R_P - R_{NP} = 400$$

The model is as follows

$$R_{kt} = \beta_0 + \beta_1 DP_{kt} + u_k + \varepsilon_{kt} \quad (3.4.1)$$

In equation (3.4.1),  $R_{kt}$  denotes the seller's revenue for round  $t$  in session  $k$ , where  $k = \{1, 2, 3, 4\}$ ,  $t = \{1, 2, \dots, 20\}$  and the dummy variable  $DP_{kt} = \begin{cases} 0, & \text{Treatment NP} \\ 1, & \text{Treatment P} \end{cases} \cdot \beta_0$  and  $\beta_1$  are the parameters to be estimated.  $\varepsilon_{kt}$  is an error term with mean zero and variance  $\sigma_\varepsilon^2$ ;  $u_k$  is the group-specific term. Table 3.4.3 shows the estimation results. From the Wald test of the estimated coefficients of  $\beta_0$  and  $\beta_1$ , we have to reject  $H1_0$  and accept the alternative hypothesis that the revenue in the NP treatment is significantly greater than predicted (p-value= 0.003). At the same time, we fail to reject  $H2_0$  which indicates that the revenue difference between the two treatments is as we predicted from the RNNE model (p-value= 0.176).



Table 3.4.3 Coefficients of pooled OLS regression

| Independent Variable | Dependent variable: Revenue |
|----------------------|-----------------------------|
| Intercept            | 2003.06*<br>(49.98)         |
| DP                   | 303.59*<br>(63.97)          |
| #Observations        | 160                         |

*Note: robust standard errors are shown in parentheses; \* significant at 5%.*

**Result (revenue):**  $R_{NP} > 1777.4$

$$R_P - R_{NP} = 400$$

### 3.4.4 Independence of bids to past prices

According to the risk neutral bidding strategy derived from Section 3.2, the bids in stage  $t$  ( $t > 1$ ) should be unrelated to the price observed in the previous stage  $t - 1$ . We test this hypothesis by regressing stage  $t$  bids on stage  $t - 1$  prices splitting the private values into four equal segments as in Section 3.4.1. The model is as follows

$$Bid_t = \beta_0 + \beta_1 Price_{t-1} + \beta_2 (PV_2 \times Price_{t-1}) + \beta_3 (PV_3 \times Price_{t-1}) + \beta_4 (PV_4 \times Price_{t-1}) + e_0$$

Where

- $PV_2 = 1$  for the first half of the intermediate value bidders, and zero otherwise;
- $PV_3 = 1$  for the second half of the intermediate value bidders, and zero otherwise;
- $PV_4 = 1$  for the high value bidders, and zero otherwise.

Therefore, for the private value categories: low value, first half of the intermediate value, second half of the intermediate value, and high value, the exact dependence of the bid on

the past price is given by  $\beta_0 + \beta_1$ ,  $\beta_0 + \beta_1 + \beta_2$ ,  $\beta_0 + \beta_1 + \beta_3$ , and  $\beta_0 + \beta_1 + \beta_4$  respectively. The estimation outcomes are reported in Table 3.4.4.

Table 3.4.4 Bids' dependence to past prices

| Treatment               | P                  |                    |                    | AP                 |                    |                    | NP                 |                    |                    |
|-------------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|--------------------|
| Unit                    | Bid <sub>t=2</sub> | Bid <sub>t=3</sub> | Bid <sub>t=4</sub> | Bid <sub>t=2</sub> | Bid <sub>t=3</sub> | Bid <sub>t=4</sub> | Bid <sub>t=2</sub> | Bid <sub>t=3</sub> | Bid <sub>t=4</sub> |
| 1×Price <sub>t-1</sub>  | -0.21              | <b>-0.08</b>       | -0.09              | -0.13              | <b>-0.01</b>       | <b>0.01</b>        | -0.25              | -0.18              | <b>-0.09</b>       |
| 2× Price <sub>t-1</sub> | <b>0.09</b>        | 0.28               | 0.35               | 0.28               | 0.44               | 0.52               | 0.13               | 0.25               | 0.41               |
| 3× Price <sub>t-1</sub> | 0.28               | 0.50               | 0.56               | 0.49               | 0.66               | 0.79               | 0.36               | 0.52               | 0.71               |
| 4× Price <sub>t-1</sub> | 0.37               | 0.57               | 0.69               | 0.58               | 0.75               | 0.92               | 0.42               | 0.58               | 0.82               |
| # Obs                   | 560                | 480                | 400                | 560                | 480                | 400                | 560                | 480                | 400                |
| Adj. R <sup>2</sup>     | 0.50               | 0.57               | 0.72               | 0.72               | 0.80               | 0.83               | 0.70               | 0.78               | 0.82               |

*Note: Bold figures indicate that the estimates are not significantly different from 0 at  $\alpha = 0.05$ . '1', '2', '3' and '4' are still the estimated slopes for the four segments [1,250], (250,500], (500,750], and (750,1000] respectively.*

If the bids are independent from previous prices as per the RNNE prediction, then each coefficient in Table 3.4.4 should be zero. However, for all three treatments, except for the first segment, the correlation between bids and previous prices is mostly significantly positive, which violates the independence hypothesis. As a result, since it is known that the bids relate to the previous prices, it raises two questions: one is whether the correlation changes or not for the last three units; the other is whether the correlation is the same for all the four segments.

Answering the above questions, we can observe from Table 3.4.4 that as the auction unravels from the second to the last stage, the correlation becomes stronger. In addition, comparing the four segments, the correlation of bids to the previous prices goes up as private value increases.

**Result (bid dependency):** In all three treatments, the bids are dependent on the previous prices. The dependency becomes stronger as the auction unravels and as private value increases.

We are also interested in whether the extent of bid dependency on previous prices differs across the three treatments. Since the bidders in the P and AP treatments know that they have to pay the penalty if they fail to obtain an item, we conjecture that if the bids depend on previous prices, then the dependency would be stronger in the P and AP treatments. From Table 3.4.4, we can observe that compared to the NP treatment, indeed the bid dependency is stronger for the AP treatment. However, the dependency is the weakest for the P treatment.

**Result (bid dependency across treatments):**  $AP > NP > P$

There are examples of other literature also identifying this bid dependent behaviour in sequential auctions. For example, Neugebauer (2004) finds that more than half of the bidding strategies exhibit price dependence. Besides this, Neugebauer and Pezanis-Christou (2007) find such an ‘anchor’ phenomenon in both the supply certainty and uncertainty treatments. Moreover, they also report that the high value bidders ‘anchor’ more than the low value bidders do, which is in line with our finding. Therefore, by re-analysing the data from Keser and Olson’s (1996) sequential auction, we provide broad support for this bid dependence phenomenon.

### 3.4.5 Allocation efficiency

From the previous sections, it is known that in the P treatment, the bidding behaviour is not as we predicted in several ways. Firstly, it appears to be non-monotonic increasing for all four units. Secondly, it relates to the price of the previous unit. Under these two conditions, naturally we would doubt whether the allocation is efficient as RNNE predicts.

Keser and Olson (1996) have reported a 98% allocation efficiency for all three treatments. However, this extremely high efficiency is due to the measurement method they use. They treat the sequential auction as a simultaneous multi-unit auction for computing the efficiency. That is, if the four bidders with the highest four private values win the objects then as a whole the allocation efficiency is 100%. The drawback of this approach to decide whether a sequential auction is efficient or not is neglecting an important feature of sequential auctions - the selling of the four objects does not occur simultaneously. Therefore, in this section, following the same method as Neugebauer and Pezanis-Christou (2007), who measure the efficiency in terms of the proportion of allocations to bidders whose value ranking was lower than or equal to the order of units were offered, the allocation efficiency for sequential auctions is computed as follows

$$Efficiency_k = \begin{cases} 100\%, & k_{th} \text{ winner's } pv > k_{th} \text{ highest } pv \\ \frac{unit\ k_{th} \text{ winner's } pv}{k_{th} \text{ highest } pv}, & \text{else} \end{cases}$$

We use an example to illustrate the first line in the equation. Suppose that the bidder with the highest private value wins the second unit. Apparently, this second unit auction allocation is

efficient, but surely it is not so for the first unit. For each unit in a treatment, we report the mean efficiency of the four sessions in Table 3.4.5.

It would be expected that the last unit auction is the most efficient, since the requirement of the last unit auction being efficient is the most lenient which is the winner is one of the top four highest value bidders. For the same reason, the first unit auction should be the least efficient one. From Table 3.4.5, we can see that the mean value of efficiency for the last unit is indeed the highest for all the treatments. However, just relying on the statistic of mean values is not particularly rigorous. Therefore, we also use a nonparametric Page test to verify this hypothesis. Let  $e_k$  be the population median for the  $k_{th}$  unit efficiency. Then we write the null hypothesis as

$$H_0 : e_1 = e_2 = e_3 = e_4$$

And the alternative hypothesis we want to verify as

$$H_1 : e_1 \leq e_2 \leq e_3 \leq e_4$$

The result of the Page test reveals a monotonic trend for the inefficiency ( $p < 0.01$ ) for all three treatments, which means at least one of the differences in  $H_1$  is a strict inequality. Furthermore, post hoc Dunn's testing shows that in all three treatments, the last unit is significantly more efficient than the first three units. In addition to that, in the NP treatment, the efficiencies of the four units have a significant increasing trend, whereas in the AP treatment, the efficiency of the second unit is greater than the first unit, but we cannot reject that the efficiencies for the second and the third units are the same. Figure 3.4.4 visually presents the allocation efficiency percentage for each unit in all treatments. It is noted that in the supply

certainty treatment reported by Neugebauer and Pezanis-Christou (2007), they find a relatively lower allocation efficiency for the first two units compared to the supply uncertainty treatment, which implies that high value bidders tend to ‘wait and see’ regarding sales of early units, thus giving low value bidders a chance to win these early units in the supply certainty treatment. In this chapter, all the three treatments have no supply uncertainty and also support the existence of the ‘wait and see’ phenomenon, as was observed from the relative inefficiency for the first three units.

Table 3.4.5 Mean efficiency by treatments

| Treatment | Unit 1 | Unit 2 | Unit 3 | Unit 4 |
|-----------|--------|--------|--------|--------|
| P         | 92.30% | 89.70% | 89.70% | 96.20% |
| AP        | 87.36% | 94.49% | 95.08% | 98.49% |
| NP        | 88.10% | 91.60% | 93.60% | 97.90% |

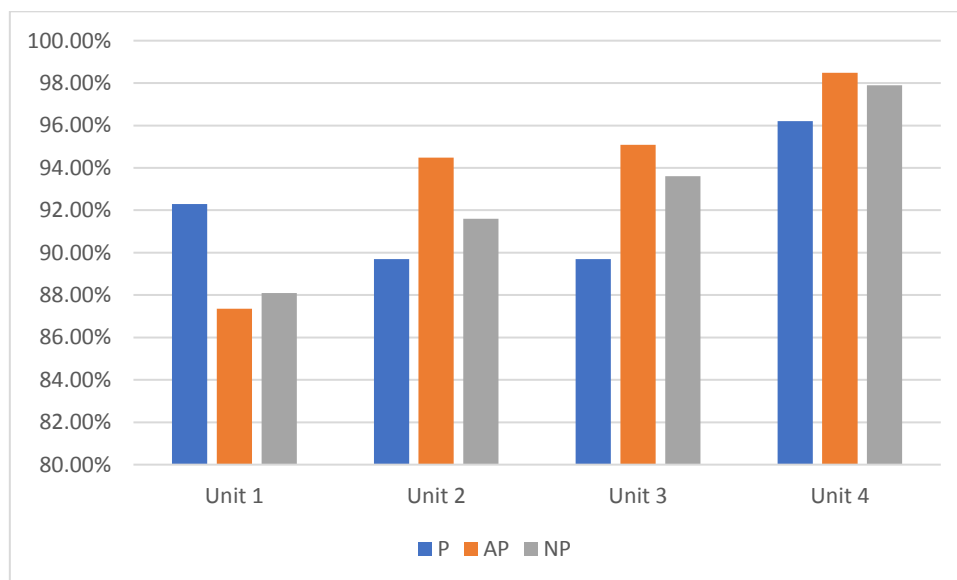


Figure 3.4.4 Percentage of efficient allocations

**Result (efficiency ranking):**

$$e_p^1 = e_p^2 = e_p^3 < e_p^4$$

$$e_{AP}^1 < e_{AP}^2 = e_{AP}^3 < e_{AP}^4$$

$$e_{NP}^1 < e_{NP}^2 < e_{NP}^3 < e_{NP}^4$$

After identifying the allocation efficiencies for the three treatments, we conduct a cross-treatment analysis to compare across the three treatments. We conjecture that in the P and AP treatments, the high value bidders would have a stronger incentive to obtain an item to avoid paying the penalty compared to the NP treatment, so if that is the case, the efficiency in these two treatments should be higher. However, from Table 3.4.5 and Figure 3.4.4, it is observed that the mean efficiency is not very different among these three treatments. When we use a stricter nonparametric Kruskal-Wallis test, we find that there is no significant difference in the efficiencies among these three treatments (p-value= 0.515).

**Result (across treatments):**  $e_p = e_{AP} = e_{NP}$

### 3.5 Conclusion

This chapter provides the deviation of the RNNE bidding strategy for each stage in sequential auctions with private values and single-unit demand with the presence of a penalty and analyses the effects of the penalty in such auctions. The data we use is from Keser and Olson (1996) and includes three treatments: No-penalty, Penalty, and Agent.

By re-examining the Penalty treatment, we identify some results that Keser and Olson (1996) do not report in their paper, such as:

- By estimating the piecewise linear bidding functions for each stage, we show that the bidding strategies are actually non-monotonic instead of concave increasing, and also depend on the previous unit's price;
- By computing the allocation efficiency on the basis of each stage instead of treating them as a whole, we find that the first three units are relatively less efficient than the last unit, which suggests the high value bidders use a 'wait-and-see' bidding strategy while giving the low value bidders some chance to win the early units; and
- The seller's revenue is greater compared to the No-penalty treatment.

Although the penalty is not designed to enhance the seller's revenue, it still provides us with some insights to consider along with the findings in Chapter 1 in relation to the payback scheme. The payback scheme increases the seller's revenue only if we determine a proper amount of initial capital balance  $K$  relative to the maximum private value; otherwise, even though the bids increase, it cannot offset the amount  $K$  retained by the winner, which results in revenue decreasing. With regards to the penalty treatment, the revenue always increases if the realised bids increase. At the same time, both the payback scheme and the penalty device indicate that when the bidders face a monetary loss, they would not respond as much as predicted.

Other features need to be taken into consideration to further analyse the effects of a penalty in sequential auctions, such as budget constraint and loss aversion. However, it is believed that by providing a detailed model and analysing the treatment effects of a penalty, this chapter can shed some light on future research for sequential auctions.



## Bibliography

- Abdellaoui, M., Bleichrodt, H., & Paraschiv, C. (2007). Loss aversion under prospect theory: A parameter-free measurement. *Management Science*, 53(10), 1659-1674.
- Abdellaoui, M., Bleichrodt, H., & l'Haridon, O. (2008). A tractable method to measure utility and loss aversion under prospect theory. *Journal of Risk and uncertainty*, 36(3), 245.
- Anderson, L. R., & Mellor, J. M. (2009). Are risk preferences stable? Comparing an experimental measure with a validated survey-based measure. *Journal of Risk and Uncertainty*, 39(2), 137-160.
- Armantier, O., & Treich, N. (2009). Subjective probabilities in games: An application to the overbidding puzzle. *International Economic Review*, 50(4), 1079-1102.
- Ashenfelter, O. (1989). How auctions work for wine and art. *The Journal of Economic Perspectives*, 3(3), 23-36.
- Becker, G. M., DeGroot, M. H., & Marschak, J. (1964). Measuring utility by a single- response sequential method. *Systems Research and Behavioral Science*, 9(3), 226-232.
- Berg, J., Dickhaut, J., & McCabe, K. (2005). Risk preference instability across institutions: A dilemma. *Proceedings of the National Academy of Sciences of the United States of America*, 102(11), 4209-4214.
- Black, J., & De Meza, D. (1992). Systematic price differences between successive auctions are no anomaly. *Journal of Economics & Management Strategy*, 1(4), 607-628.
- Bortolotti, B. (2001). *Privatisation, large shareholders, and sequential auctions of shares* (No. 37.2001). Nota di Lavoro, Fondazione Eni Enrico Mattei.
- Buccola, S. T. (1982). Price trends at livestock auctions. *American Journal of Agricultural Economics*, 64(1), 63-69.
- Burns, P. (1985). *Experience and decision making: A comparison of students and businessmen in a simulated progressive auction* (No. 00135). The Field Experiments Website.
- Chen, K. Y., & Plott, C. R. (1998). Nonlinear behavior in sealed bid first price auctions. *Games and Economic Behavior*, 25(1), 34-78.
- Concina, L. (2014). *Risk attitude & economics*. FonCSI. Retrieved from <https://www.foncsi.org/fr/publications/collections/regards/risk-attitude-and-economics/Viewpoint-risk-attitude-economics.pdf>
- Coppinger, V. M., Smith, V. L., & Titus, J. A. (1980). Incentives and Behavior in English, Dutch and Sealed- Bid Auctions. *Economic inquiry*, 18(1), 1-22.

- Cox, J. C., Roberson, B., & Smith, V. L. (1982). Theory and behavior of single object auctions. *Research in experimental economics*, 2(1), 1-43.
- Cox, J. C., Smith, V. L., & Walker, J. M. (1982). Auction market theory of heterogeneous bidders. *Economics Letters*, 9(4), 319-325.
- Cox, J. C., Smith, V. L., & Walker, J. M. (1983). Tests of a heterogeneous bidders theory of first price auctions. *Economics Letters*, 12(3-4), 207-212.
- Cox, J. C., Smith, V. L., & Walker, J. M. (1984). Theory and behavior of multiple unit discriminative auctions. *The Journal of Finance*, 39(4), 983-1010.
- Cox, J. C., Smith, V. L., & Walker, J. M. (1985). Experimental development of sealed-bid auction theory; calibrating controls for risk aversion. *The American Economic Review*, 75(2), 160-165.
- Cox, J. C., Smith, V. L., & Walker, J. M. (1988). Theory and individual behavior of first-price auctions. *Journal of Risk and uncertainty*, 1(1), 61-99.
- Cox, J. C., Smith, V. L., & Walker, J. M. (1992). Theory and misbehavior of first-price auctions: Comment. *The American Economic Review*, 82(5), 1392-1412.
- Cox, J. C., & Oaxaca, R. L. (1996). Is bidding behavior consistent with bidding theory for private value auctions? *Research in experimental economics*, 6, 131-148.
- Crawford, V. P., & Iriberri, N. (2007). Level- k Auctions: Can a Nonequilibrium Model of Strategic Thinking Explain the Winner's Curse and Overbidding in Private- Value Auctions? *Econometrica*, 75(6), 1721-1770.
- Dorsey, R., & Razzolini, L. (2003). Explaining overbidding in first price auctions using controlled lotteries. *Experimental Economics*, 6(2), 123-140.
- Engelbrecht-Wiggans, R., & Katok, E. (2008). Regret and feedback information in first-price sealed-bid auctions. *Management Science*, 54(4), 808-819.
- Delgado, M. R., Schotter, A., Ozbay, E. Y., & Phelps, E. A. (2008). Understanding overbidding: using the neural circuitry of reward to design economic auctions. *Science*, 321(5897), 1849-1852.
- Dinno, A. (2015). Nonparametric pairwise multiple comparisons in independent groups using Dunn's test. *Stata Journal*, 15(1), 292-300.
- Dorsey, R. E. (1989). *The effects of endogenously-generated information on specific economic institutions*. Published PhD thesis. University of Arizona – Tucson.

- Dorsey, R., & Razzolini, L. (2003). Explaining overbidding in first price auctions using controlled lotteries. *Experimental Economics*, 6(2), 123-140.
- Dyer, D., Kagel, J. H., & Levin, D. (1989). Resolving uncertainty about the number of bidders in independent private-value auctions: an experimental analysis. *The RAND Journal of Economics*, 268-279.
- Easley, D., & Kleinberg, J. (2010). *Networks, crowds, and markets: Reasoning about a highly connected world*. Cambridge University Press.
- Engel, R. P. (2007). *Essays on risk and incentives*. Published PhD Thesis. The Florida State University. Retrieved from <http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.879.1119&rep=rep1&type=pdf>
- Fehr, E., & Goette, L. (2007). Do workers work more if wages are high? Evidence from a randomized field experiment. *The American Economic Review*, 97(1), 298-317.
- Filiz, E., & Ozbay, E. Y. (2007). Auctions with anticipated regret: Theory and experiment. *American Economic Review*, 97(4), 1407-1418.
- Fiorio, C. V. (2004). Confidence intervals for kernel density estimation. *Stata Journal*, 4, 168-179.
- Fischbacher, U. (2007). z-Tree: Zurich toolbox for ready-made economic experiments. *Experimental economics*, 10(2), 171-178.
- Gächter, S., Johnson, E. J., & Herrmann, A. (2007). Individual-level loss aversion in riskless and risky choices. Retrieved from <http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.337.2855&rep=rep1&type=pdf>
- Goeree, J. K., Holt, C. A., & Palfrey, T. R. (2002). Quantal response equilibrium and overbidding in private-value auctions. *Journal of Economic Theory*, 104(1), 247-272.
- Goldstein, D. G., Johnson, E. J., & Sharpe, W. F. (2008). Choosing outcomes versus choosing products: Consumer-focused retirement investment advice. *Journal of Consumer Research*, 35(3), 440-456.
- Greiner, B. (2015). Subject pool recruitment procedures: organizing experiments with ORSEE. *Journal of the Economic Science Association*, 1(1), 114-125.

- Harris, M., & Raviv, A. (1981). Allocation mechanisms and the design of auctions. *Econometrica: Journal of the Econometric Society*, 1477-1499.
- Harrison, G. W. (1986). An experimental test for risk aversion. *Economics Letters*, 21(1), 7-11.
- Harrison, G. W. (1989). Theory and misbehavior of first-price auctions. *The American Economic Review*, 749-762.
- Harrison, G. W. (1990). Risk attitudes in first-price auction experiments: A Bayesian analysis. *The Review of Economics and Statistics*, 541-546.
- Harrison, G. W., Johnson, E., McInnes, M. M., & Rutström, E. E. (2005). Risk aversion and incentive effects: Comment. *American Economic Review*, 897-901.
- Hausch, D. B. (1986). Multi-object auctions: Sequential vs. simultaneous sales. *Management Science*, 32(12), 1599-1610.
- Hey, J. D., Morone, A., & Schmidt, U. (2009). Noise and bias in eliciting preferences. *Journal of Risk and Uncertainty*, 39(3), 213-235.
- Holt Jr, C. A. (1980). Competitive bidding for contracts under alternative auction procedures. *Journal of political Economy*, 88(3), 433-445.
- Holt, C. A., & Laury, S. (2002). Risk aversion and incentive effects. Retrieved from <http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.38.4218&rep=rep1&type=pdf>
- Hu, A., Offerman, T., & Onderstal, S. (2011). Fighting collusion in auctions: an experimental investigation. *International Journal of Industrial Organization*, 29(1), 84-96.
- Hu, A., & Zou, L. (2015). Sequential auctions, price trends, and risk preferences. *Journal of Economic Theory*, 158, 319-335.
- Isaac, R. M., & Walker, J. M. (1985). Information and conspiracy in sealed bid auctions. *Journal of Economic Behavior & Organization*, 6(2), 139-159.
- Isaac, R. M., & James, D. (2000). Just who are you calling risk averse?. *Journal of Risk and Uncertainty*, 20(2), 177-187.
- Jeitschko, T. D. (1999). Equilibrium price paths in sequential auctions with stochastic supply. *Economics Letters*, 64(1), 67-72.
- Kagel, J. H., & Roth, A. E. (1992). Theory and misbehavior in first-price auctions: comment. *The American economic review*, 82(5), 1379-1391.
- Kagel, J. H., & Levin, D. (1993). Independent private value auctions: Bidder behaviour in first-, second-and third-price auctions with varying numbers of bidders. *The Economic Journal*, 103(419), 868-879.

- Kagel, J. H., & Roth, A. E. (1995). *The handbook of experimental economics*. Princeton: Princeton University Press.
- Kagel, J., & Levin, D. (2011). Auctions: A Survey of Experimental Research, 1995-2010. forthcoming in *The Handbook of Experimental Economics*, Vol. 2.
- Kahneman, D., & Tversky, A. (1979). Prospect theory: An analysis of decision under risk. *Econometrica: Journal of the Econometric Society*, 263-291.
- Kahneman, D., Knetsch, J. L., & Thaler, R. H. (1990). Experimental tests of the endowment effect and the Coase theorem. *Journal of Political Economy*, 98(6), 1325-1348.
- Kahneman, D., Knetsch, J. L., & Thaler, R. H. (1991). Anomalies: The endowment effect, loss aversion, and status quo bias. *The journal of economic perspectives*, 5(1), 193-206.
- Kahneman, D., & Lovallo, D. (1993). Timid choices and bold forecasts: A cognitive perspective on risk taking. *Management Science*, 39(1), 17-31.
- Keser, C., & Olson, M. (1996). Experimental examination of the declining price anomaly. In V. Ginsburgh and P.M. Menger (Eds.), *Economics of the Arts: Selected Essays* (pp. 151–76). Amsterdam: North Holland.
- Kocher, M. G., Pahlke, J., & Trautmann, S. T. (2010). *An experimental test of precautionary bidding* (No. 2010-30). Munich Discussion Paper.
- Kocher, M. G., Pahlke, J., & Trautmann, S. T. (2015). An experimental test of precautionary bidding. *European Economic Review* 78, 27-38.
- Kőszegi, B., & Rabin, M. (2006). A model of reference-dependent preferences. *The Quarterly Journal of Economics*, 121(4), 1133-1165.
- Krishna, V. (2009). *Auction theory*. Cambridge, MA: Academic Press.
- Lange, A., & Ratan, A. (2010). Multi-dimensional reference-dependent preferences in sealed-bid auctions—How (most) laboratory experiments differ from the field. *Games and Economic Behavior*, 68(2), 634-645.
- Laury, S., & Holt, C. A. (2005). *Further reflections on prospect theory*. Andrew Young School of Policy Studies Research Paper Series, (06-11).
- Liao, E. Z., & Holt, C. A. (2013). *The pursuit of revenue reduction: an experimental analysis of the Shanghai license plate auction*. Retrieved from [http://people.virginia.edu/~cah2k/shanghai\\_auction\\_paperv4.pdf](http://people.virginia.edu/~cah2k/shanghai_auction_paperv4.pdf).
- Maskin, E., & Riley, J. G. (1980). *Auctioning an indivisible object*. Tech. rep., Kennedy School, Harvard University.
- Maskin, E., & Riley, J. (2000). Asymmetric auctions. *The Review of Economic Studies*, 67(3), 413-438.

- McAfee, R. P., & Vincent, D. (1993). The declining price anomaly. *Journal of Economic Theory*, 60(1), 191-212.
- McMillan, J. (1994). Selling spectrum rights. *The Journal of Economic Perspectives*, 8(3), 145-162.
- Mezzetti, C. (2011). Sequential auctions with informational externalities and aversion to price risk: decreasing and increasing price sequences. *The Economic Journal*, 121(555), 990-1016.
- Milgrom, P.R., & Weber, R.J. (1982). A theory of auctions and competitive bidding: part 2, in: Klemperor, P. (Ed.), 2000. *The economic theory of auctions*. Cheltenham: Edward Elgar, Mimeo, Northwestern University, pp. 179–194.
- Moffatt, P. G. (2015). *Experiments: Econometrics for Experimental Economics*. London: Palgrave Macmillan.
- Neugebauer, T. (2004). Bidding strategies of sequential first price auctions programmed by experienced bidders. *Cuadernos de Economía*, 27(75), 153-184.
- Neugebauer, T., & Selten, R. (2006). Individual behavior of first-price auctions: The importance of information feedback in computerized experimental markets. *Games and Economic Behavior*, 54(1), 183-204.
- Neugebauer, T., & Pezanis-Christou, P. (2007). Bidding behavior at sequential first-price auctions with (out) supply uncertainty: A laboratory analysis. *Journal of Economic Behavior & Organization*, 63(1), 55-72.
- Neugebauer, T., & Perote, J. (2008). Bidding ‘as if’ risk neutral in experimental first price auctions without information feedback. *Experimental Economics*, 11(2), 190-202.
- Palfrey, T. R., & Pevnitskaya, S. (2008). Endogenous entry and self-selection in private value auctions: An experimental study. *Journal of Economic Behavior & Organization*, 66(3), 731-747.
- Pezanis-Christou, P. (2002). On the impact of low-balling: Experimental results in asymmetric auctions. *International Journal of Game Theory*, 31(1), 69-89.
- Pezanis-Christou, P., & Romeu, A. (2002). *Structural Inferences from First-Price Auction Experiments*. Unitat de Fonaments de l'Anàlisi Econòmica (UAB) and Institut d'Anàlisi Econòmica (CSIC).
- Pezanis-Christou, P. & Wu, H. (2014). "Loss Aversion and regret in common value auctions", mimeo, University of Adelaide.
- Prasad, K., & Salmon, T. C. (2013). Self Selection and market power in risk sharing contracts. *Journal of Economic Behavior & Organization*, 90, 71-86.

- Rabin, M. (2000). Risk aversion and expected-utility theory: A calibration theorem. *Econometrica*, 68(5), 1281-1292.
- Rosato, A. (2014). *Loss Aversion in Sequential Auctions: Endogenous Interdependence, Informational Externalities and the "Afternoon Effect"*. Retrieved from [https://mpira.ub.uni-muenchen.de/56824/1/MPRA\\_paper\\_56824.pdf](https://mpira.ub.uni-muenchen.de/56824/1/MPRA_paper_56824.pdf)
- Schram, A. J., & Onderstal, S. (2009). Bidding to give: An experimental comparison of auctions for charity. *International Economic Review*, 50(2), 431-457.
- Siegel, S. (1956). *Nonparametric statistics for the behavioral sciences*. New York: McGraw-Hill.
- Siegel, S. C., & Castellan, J. (1988). *Nonparametric statistics for the behavioural sciences*. New York: McGraw-Hill.
- Thaler, R. H., Tversky, A., Kahneman, D., & Schwartz, A. (1997). The effect of myopia and loss aversion on risk taking: An experimental test. *The Quarterly Journal of Economics*, 647-661.
- Tom, S. M., Fox, C. R., Trepel, C., & Poldrack, R. A. (2007). The neural basis of loss aversion in decision-making under risk. *Science*, 315(5811), 515-518.
- Tversky, A., & Kahneman, D. (1992). Advances in prospect theory: Cumulative representation of uncertainty. *Journal of Risk and Uncertainty*, 5(4), 297-323.
- Van Boening, M. V., Rassenti, S. J., & Smith, V. L. (1998). Numerical computation of equilibrium bid functions in a first-price auction with heterogeneous risk attitudes. *Experimental Economics*, 1(2), 147-159.
- Vickrey, W. (1961). Counterspeculation, auctions, and competitive sealed tenders. *The Journal of finance*, 16(1), 8-37.
- Walker, J. M., Smith, V. L., & Cox, J. C. (1990). Inducing risk-neutral preferences: An examination in a controlled market environment. *Journal of Risk and Uncertainty*, 3(1), 5-24.
- Weber, R. J. (1981). *Multiple-object auctions*. Northwestern University. Retrieved from <https://ksmb55a.kellogg.northwestern.edu/research/math/papers/496.pdf>

## Appendix A

### Welcome to the experiment!

You will receive a show-up payment of \$10 for participating in this experiment which consists of four independent parts. For each of these four parts, you will be given written instructions which will be read aloud.

In each part of the experiment, you will get a payoff which will depend both on your decisions and on chance.

To determine your earnings for participating in this experiment, the payoffs you get in these four parts will be added to your show-up payment of \$10.

Note that in Part 1, your payoff may be positive or negative. Positive payoffs are added to your show-up fee of \$10 whereas negative payoffs are subtracted from it.

You are not allowed to communicate with other participants during the experiment.

If you have any questions, please raise your hand and someone will answer your questions individually.

If you have no questions, then please proceed to reading the instructions for Part 1.

### Part 1: lotteries

In this first part, you are asked to answer fourteen questions. Each of these questions asks whether you want to participate or not in a lottery which yields a **win** (in dollars) with **50%** chance and a **loss** (in dollars) with **50%** chance. If you would like to participate in the proposed lottery, please select “Yes” or if you do not wish to, then select “No”. Each of the fourteen questions relates to a different lottery for which you have to decide whether to participate or not by answering “Yes” or “No”.

**At the end of the experiment, one** of the fourteen questions will be randomly selected to determine your payoff for participating in this first part. If you answered “Yes” to the selected question, then you will participate in the selected lottery and your payoff for participating in this first part will be equal to the outcome of this lottery. If you selected “No” then you will not participate in the selected lottery and your payoff for this first part will be \$0.

**Example:** suppose that *at the end of the experiment*, question 3 is drawn. Assume that question 3 asked whether you want to participate in a lottery in which you can either win \$X with 50% chance or lose \$Y with 50% chance.



*If you answered “Yes” to question 3, then your payoff for participating in this first part will be the outcome of the proposed lottery. It can either be a gain of \$X or a loss of \$Y.*

*If you answered “No” to question 3, then your payoff for participating in this first part will be \$0.*

If you have any questions, please raise your hand and someone will answer your questions individually.

If you have no questions, then please answer the following comprehension questions. Note that your answers to these questions do not affect your earnings in any way. The purpose of these questions is just to make sure that you understand the game you are about to play.

## **Part 2: selling a lottery ticket**

In this second part, you are the owner of eleven lottery tickets. Each of these lottery tickets yields a high payoff with 50% chance or a low payoff with 50% chance, and you are given an opportunity to sell the tickets.

If you want to sell your lottery ticket, then you must state a ‘selling price’ between the low payoff and the high payoff (which can be equal to the low or high payoff). A ‘selling price’ is the minimum price at which you are willing to sell your lottery ticket.

The buyer of your lottery ticket is played by the computer which has been programmed to make a random ‘buying price’ between the low payoff and the high payoff (inclusive). This means that the computer can offer any ‘buying price’ between the low payoff and the high payoff (inclusive) with equal chance. Note that the computer’s ‘buying price’ does not depend in any way on your ‘selling price’ and that it can have up to two decimals.

**At the end of the experiment**, one of the eleven lottery tickets will be randomly selected to determine your payoff for participating in this second part.

There are two possible outcomes:

- First, your ‘selling price’ is greater than the computer’s ‘buying price’. In this case, you do not sell your lottery ticket and your payoff, which will be determined **at the end of the experiment**, will be equal to the outcome of the lottery. That is, you will either earn the high payoff with 50% chance or the low payoff with 50% chance.
- Second, your ‘selling price’ is smaller or equal to the computer’s ‘buying price’. In this case, you do sell your lottery ticket and your payoff will be equal to the computer’s ‘buying price’.

**Example:** suppose that *at the end of the experiment*, lottery 5 is drawn. Assume that in this lottery you can either gain \$7 with 50% chance or gain \$2 with 50% chance and you state \$4 as the 'selling price'.

If the computer's 'buying price' is \$5, then you sell your lottery ticket to the computer and your payoff for participating in this second part will be equal to \$5.

If, on the other hand, it turns out that the computer's 'buying price' is \$3, then you keep your lottery ticket and your payoff for participating in this second part will be equal to the outcome of the lottery, that is, you will either receive a payoff of \$7 with 50% chance or of \$2 with 50% chance.

If you have any questions, please raise your hand and someone will answer your questions individually.

If you have no questions, then please answer the following comprehension questions. Note that your answers to these questions do not affect your earnings in any way. The purpose of these questions is just to make sure that you understand the game you are about to play.

### Part 3: Auction I

In this third part, you are competing in a market with six buyers and you are one of them. The other five buyers are other participants in this lab.

You are participating in 20 auctions with the same group of buyers.

**At the beginning of each auction**, the computer will randomly determine a value for you, which may be any cent amount between and including \$0.00 and \$10.00, with each amount in this interval being equally likely to be chosen.

The values for other bidders are also randomly drawn, with each cent amount between \$0.00 and \$10.00 being equally likely.

Your value will be independent of any other buyer's value and is also independent of your own value in other auctions.

You can submit any bid up to your value (with up to two decimals). If you do not want to participate in this auction, then you must submit a bid of \$0.

If you submit the highest bid then you win the auction. In this case, you pay a price equal to your bid and get the following payoff:

|  |
|--|
| $\text{Your payoff} = \text{your value} - \text{your bid} \quad (\text{if you win})$ |
|--|

If your bid is not the highest then you lose the auction. In this case, you get the following payoff:

|                 |               |
|-----------------|---------------|
| Your payoff = 0 | (if you lose) |
|-----------------|---------------|

If there is no single highest bid, then one of the equal highest bidders will be randomly determined as the winner of the auction.

**At the end of each auction**, you will find out whether you have won or lost the auction, the payoff you have, and what is the highest bid.

**At the end of the experiment**, two of the 20 auctions will be randomly selected to determine your payoff for participating in this third part.

**Example:** suppose that **at the end of the experiment**, auctions 6 and 12 are drawn and your payoffs for the two auctions are the following:

Auction 6: \$A

Auction 12: \$B

Your earnings from this third part will be equal to:  $A+B$ .

Assume that in auction 6, you have a value of \$ 9, and you submit a bid of \$ 6.

If you are the winner, your payoff for participating in auction 6 is  $9-6 = 3$ .

If you lose the auction, your payoff for participating in auction 6 is \$0.

If you have any questions, please raise your hand and someone will answer your questions individually.

If you have no questions, then please answer the following comprehension questions. Note that your answers to these questions do not affect your earnings in any way. The purpose of these questions is just to make sure that you understand the game you are about to play.

### Part 4: Auction II

In this fourth and last part, you are participating in 20 auctions with the same group of buyers as in part 3.

The basic setting for this part is the same as in part 3, which is:

**At the beginning of each auction**, the computer will randomly determine a value for you, which may be any cent amount between and including \$0.00 and \$10.00, with each amount in this interval being equally likely to be chosen.

The values for other bidders are also randomly drawn, with each cent amount between \$0.00 and \$10.00 being equally likely.

Your value will be independent of any other buyer's value and is also independent of your own value in other auctions.

The difference in this part is:

|   |
|---|
| <b>At the beginning of each auction, you are given \$5.</b> |
|---|

Your bid can be any cent amount up to your value plus \$5. If you do not want to participate in this auction, then you must submit a bid of \$0.

If you submit the highest bid then you win the auction. In this case, you **keep the \$5** and pay a price equal to your bid. Your payoff is defined as follows:

|  |
|--|
| Your payoff = $\$5 + \text{your value} - \text{your bid}$ (if you win) |
|--|

If your bid is not the highest then you lose the auction. In this case, you must **give the \$5 back**. Your payoff is defined as follows:

|   |
|---|
| Your payoff = $\$5 - \$5 = 0$ (if you lose) |
|---|

If there is no single highest bid, then one of the equal highest bidders will be randomly determined as the winner of the auction.

**At the end of each auction**, you will find out whether you have won or lost the auction, the payoff you have, and what is the highest bid.

**At the end of the experiment**, two of the 20 auctions will be randomly selected to determine your payoff for participating in this fourth part.

*Suppose that **at the end of the experiment**, auctions 6 and 12 are drawn and your payoffs for the two auctions are the following:*

*Auction 6: \$A*

*Auction 12: \$B*

*Your earnings from this fourth part will be equal to:  $\$A + \$B$ .*

**Example 1:** Assume that in auction 6, you have a value of \$ 9, and you submit a bid of \$ 6.

*If you are the winner, you keep the \$5, so your payoff for auction 6 is  $\$5 + \$9 - \$6 = \$8$ .*

*If you lose the auction, then you must pay the \$5 back, so your payoff for auction 6 is  $\$5 - \$5 = \$0$ .*

**Example 2:** Assume that in auction 6, you have a value of \$ 9, and you submit a bid of \$ 12.

*If you are the winner, you keep the \$5, so your payoff for auction 6 is  $\$5 + \$9 - \$12 = \$2$ .*

*If you lose the auction, then you must pay the \$5 back, so your payoff for auction 6 is  $\$5 - \$5 = \$0$ .*

If you have any questions, please raise your hand and someone will answer your questions individually.

If you have no questions, then please answer the following comprehension questions. Note that your answers to these questions do not affect your earnings in any way. The purpose of these questions is just to make sure that you understand the game you are about to play.

## **Appendix B**

The following equation is the first-order differentiation equation of Risk Averse symmetric Nash equilibrium (RASNE) bidding strategy.

$$b'(v_i) = \frac{(n-1)(K + v_i - b(v_i))}{v_i r}$$

$$b'(v_i) = -\frac{(n-1)b(v_i)}{v_i r} + \frac{n-1}{r} + \frac{(n-1)K}{v_i r} \quad (\text{B.1})$$

Let  $p(v_i) = \frac{n-1}{v_i r}$ ,  $q(v_i) = \frac{n-1}{r} + \frac{(n-1)K}{v_i r}$  and they are known by the bidders.

Therefore we can write equation (B.1) as follows

$$b'(v_i) + p(v_i)b(v_i) = q(v_i)$$

Multiplying each side by an integrating factor  $m(v)$ , which yields

$$m(v)b'(v_i) + m(v)p(v_i)b(v_i) = m(v)q(v_i) \quad (\text{B.2})$$

And also in particular we require

$$m(v)b'(v_i) + m(v)p(v_i)b(v_i) = [m(v)b(v_i)]'$$

So, equation (B.2) becomes

$$[m(v)b(v_i)]' = m(v)q(v_i) \quad (\text{B.3})$$

Which implies that  $m'(v) = p(v_i)m(v)$

We know that  $m'(v) = \frac{dm}{dv}$ , so  $\frac{dm}{dv} = p(v_i)m(v)$

$$\frac{dm}{m} = p(v_i)dv$$

Integrating both sides y

$$\ln m(v) = \int_0^v p(t)dt$$

$$m(v) = \exp\left[\int_0^v p(t)dt\right] = \exp\left[\frac{n-1}{r} \int_0^v \frac{1}{t} dt\right] = \exp\left[\frac{n-1}{r} \cdot \ln v\right] = (e^{\ln v})^{\frac{n-1}{r}} = v^{\frac{n-1}{r}}$$

Integrating both sides for equation (B.3)

$$m(v)b(v_i) = \int_0^v m(t)q(t)dt$$

$$b(v_i) = \frac{1}{m(v)} \int_0^v m(t)q(t)dt$$

$$\begin{aligned} &= \frac{1}{v^{\frac{n-1}{r}}} \int_0^v t^{\frac{n-1}{r}} \left( \frac{n-1}{r} + \frac{(n-1)K}{tr} \right) dt \\ &= \frac{1}{v^{\frac{n-1}{r}}} \left[ \frac{n-1}{r} \int_0^v t^{\frac{n-1}{r}} dt + \frac{(n-1)K}{r} \int_0^v t^{\frac{n-1}{r}-1} dt \right] \\ &= \frac{1}{v^{\frac{n-1}{r}}} \left[ \frac{n-1}{r} \cdot \frac{v^{\frac{n-1}{r}+1}}{\frac{n-1}{r}+1} + \frac{(n-1)K}{r} \frac{v^{\frac{n-1}{r}}}{\frac{n-1}{r}} \right] \\ &= \frac{n-1}{n-1+r} v + K \end{aligned}$$

## Appendix C

The estimated risk parameter for each subject in the cases of  $r_{max}=1$  and  $r_{max}=2$

| Series | Session,<br>(# Auctions) | Subject | $r_{max}=1$ |                |               |        |       | $r_{max}=2$ |                |               |        |       |
|--------|--------------------------|---------|-------------|----------------|---------------|--------|-------|-------------|----------------|---------------|--------|-------|
|        |                          |         | $R^2$       | $\hat{\alpha}$ | $\hat{\beta}$ | # obs. | $r$   | $R^2$       | $\hat{\alpha}$ | $\hat{\beta}$ | # obs. | $r$   |
| 1(3)   | dfd10', (10)             | 1       | 0.993       | -0.019         | 0.704**       | 7      | 0.841 | 0.991       | -0.026         | 0.727**       | 6      | 0.751 |
|        |                          | 2       | 0.953       | -0.027         | 0.765**       | 9      | 0.614 | 0.927       | -0.01          | 0.668**       | 6      | 0.994 |
|        |                          | 3       | 0.902       | 0.02           | 0.664**       | 9      | 1.012 | 0.877       | 0.033          | 0.574**       | 6      | 1.484 |
|        | dfd3, (10)               | 1       | 0.603       | 0.1            | 0.461**       | 9      | 2.338 | 0.603       | 0.1            | 0.461**       | 9      | 2.338 |
|        |                          | 2       | 0.18        | 0.171          | 0.314         | 10     | n.a.  | 0.081       | 0.192          | 0.27          | 9      | n.a.  |
|        |                          | 3       | 0.123       | 0.188          | 0.297         | 8      | n.a.  | 0.096       | 0.231          | 0.13          | 5      | n.a.  |
|        | fdf10, (20)              | 1       | 0.949       | -0.03          | 0.738**       | 17     | 0.710 | 0.925       | -0.041         | 0.763**       | 14     | 0.621 |
|        |                          | 2       | 0.891       | 0              | 0.644**       | 16     | 1.106 | 0.81        | -0.005         | 0.653**       | 12     | 1.063 |
|        |                          | 3       | 0.929       | -0.021         | 0.767**       | 17     | 0.608 | 0.933       | 0.005          | 0.696**       | 16     | 0.874 |
|        | fdf3', (20)              | 1       | 0.836       | 0.041          | 0.609**       | 17     | 1.284 | 0.805       | 0.06           | 0.545**       | 14     | 1.670 |
|        |                          | 2       | 0.96        | 0.014          | 0.658**       | 18     | 1.040 | 0.921       | 0.018          | 0.636**       | 15     | 1.145 |
|        |                          | 3       | 0.839       | 0.01           | 0.534**       | 14     | 1.745 | 0.849       | 0.026          | 0.487**       | 13     | 2.107 |
|        |                          |         |             |                |               |        |       |             |                |               |        |       |
|        |                          |         |             |                |               |        |       |             |                |               |        |       |
|        |                          |         |             |                |               |        |       |             |                |               |        |       |
| 1'(3)  | fpn3(1), (20)            | 1       | 0.934       | -0.025         | 0.807**       | 19     | 0.478 | 0.933       | -0.006         | 0.723**       | 15     | 0.766 |
|        |                          | 2       | 0.984       | 0.008          | 0.881**       | 15     | 0.270 | 0.953       | 0.001          | 0.904**       | 11     | 0.212 |
|        |                          | 3       | 0.98        | 0.004          | 0.723**       | 18     | 0.766 | 0.975       | 0.008          | 0.7**         | 15     | 0.857 |
|        |                          |         |             |                |               |        |       |             |                |               |        |       |
|        |                          | 1       | 0.993       | -0.037**       | 0.89**        | 19     | n.a.  | 0.986       | -0.041**       | 0.906**       | 14     | n.a.  |



|  |  |   |       |        |         |    |       |       |       |         |    |       |
|--|--|---|-------|--------|---------|----|-------|-------|-------|---------|----|-------|
|  | f <sub>p</sub> n <sub>3</sub> (2),<br>(20) | 2 | 0.943 | -0.032 | 0.889** | 16 | 0.250 | 0.829 | 0.007 | 0.767** | 11 | 0.608 |
|  |  | 3 | 0.989 | 0.011  | 0.873** | 16 | 0.291 | 0.997 | 0     | 0.932** | 14 | 0.146 |

|      |                |   |       |          |         |    |       |       |          |         |    |       |
|------|----------------|---|-------|----------|---------|----|-------|-------|----------|---------|----|-------|
| 2(4) | dfd8', (10)    | 1 | 0.992 | -0.009   | 0.908** | 10 | 0.304 | 0.994 | -0.021   | 0.938** | 9  | 0.198 |
|      |                | 2 | 0.992 | -0.023   | 0.911** | 9  | 0.293 | 0.998 | -0.011   | 0.884** | 6  | 0.394 |
|      |                | 3 | 0.994 | -0.028   | 0.913** | 9  | 0.286 | 0.987 | -0.029   | 0.917** | 7  | 0.272 |
|      |                | 4 | 0.978 | -0.052   | 0.963** | 8  | 0.115 | 0.977 | -0.04    | 0.911** | 7  | 0.398 |
|      |                |   |       |          |         |    |       |       |          |         |    |       |
|      | fdf8, (20)     | 1 | 0.995 | -0.034** | 0.955** | 14 | n.a.  | 0.995 | -0.027** | 0.928** | 12 | n.a.  |
|      |                | 2 | 0.985 | -0.025   | 0.912** | 15 | 0.289 | 0.988 | -0.02    | 0.892** | 14 | 0.405 |
|      |                | 3 | 0.986 | -0.029** | 0.882** | 15 | n.a.  | 0.989 | -0.031** | 0.879** | 12 | n.a.  |
|      |                | 4 | 0.98  | -0.054** | 0.837** | 19 | n.a.  | 0.986 | -0.055** | 0.832** | 15 | n.a.  |
|      |                |   |       |          |         |    |       |       |          |         |    |       |
| 4(4) | fplonci1, (25) | 1 | 0.992 | -0.02**  | 0.956** | 19 | n.a.  | 0.981 | -0.008   | 0.895** | 14 | 0.352 |
|      |                | 2 | 0.929 | -0.032   | 0.895** | 25 | 0.352 | 0.875 | -0.041   | 0.919** | 19 | 0.264 |
|      |                | 3 | 0.957 | -0.013   | 0.906** | 23 | 0.311 | 0.907 | -0.011   | 0.889** | 17 | 0.375 |
|      |                | 4 | 0.972 | -0.037** | 0.925** | 20 | n.a.  | 0.967 | -0.044** | 0.952** | 19 | n.a.  |
|      |                |   |       |          |         |    |       |       |          |         |    |       |
|      | fplonci2, (25) | 1 | 0.579 | 0.084    | 0.665** | 20 | 1.511 | 0.217 | 0.139    | 0.35    | 14 | n.a.  |
|      |                | 2 | 0.988 | 0.02     | 0.887** | 25 | 0.382 | 0.999 | -0.007** | 0.981** | 17 | n.a.  |
|      |                | 3 | 0.918 | -0.03    | 0.994** | 20 | 0.018 | 0.835 | -0.027   | 0.980** | 16 | 0.061 |
|      |                | 4 | 0.998 | 0.001    | 0.95**  | 20 | 0.158 | 0.998 | 0        | 0.954** | 19 | 0.145 |
|      |                |   |       |          |         |    |       |       |          |         |    |       |
|      | fplonci3, (25) | 1 | 0.971 | 0.012    | 0.851** | 22 | 0.525 | 0.975 | 0.003    | 0.872** | 15 | 0.440 |
|      |                | 2 | 0.962 | -0.041** | 0.907** | 25 | n.a.  | 0.958 | -0.058** | 0.963** | 21 | n.a.  |
|      |                | 3 | 0.97  | 0.038**  | 0.806** | 24 | n.a.  | 0.962 | 0.033    | 0.818** | 19 | 0.667 |
|      |                | 4 | 0.786 | 0.09**   | 0.577** | 22 | n.a.  | 0.786 | 0.090**  | 0.577** | 22 | n.a.  |
|      |                |   |       |          |         |    |       |       |          |         |    |       |
|      | fplonci4, (25) | 1 | 0.998 | -0.004   | 0.975** | 19 | 0.077 | 0.995 | -0.001   | 0.959** | 14 | 0.128 |
|      |                | 2 | 0.994 | 0.006    | 0.935** | 23 | 0.209 | 0.998 | -0.005   | 0.978** | 16 | 0.067 |
|      |                | 3 | 0.997 | 0.015**  | 0.917** | 21 | n.a.  | 0.995 | 0.011    | 0.930** | 16 | 0.226 |
|      |                | 4 | 0.462 | 0.095    | 0.494** | 13 | 3.073 | 0.363 | 0.108    | 0.451** | 12 | 3.652 |

|  |                 |   |       |         |         |    |       |       |          |         |    |       |
|--|-----------------|---|-------|---------|---------|----|-------|-------|----------|---------|----|-------|
|  |                 |   |       |         |         |    |       |       |          |         |    |       |
|  | fplonci5, (25)  | 1 | 0.975 | 0.007   | 0.894** | 21 | 0.356 | 0.967 | -0.011   | 0.96**  | 14 | 0.125 |
|  |                 | 2 | 0.994 | -0.001  | 0.951** | 21 | 0.155 | 0.991 | -0.009   | 0.980** | 16 | 0.061 |
|  |                 | 3 | 0.999 | 0.006** | 0.97**  | 20 | n.a.  | 0.999 | 0.002    | 0.982** | 16 | 0.055 |
|  |                 | 4 | 0.984 | 0.025** | 0.818** | 24 | n.a.  | 0.996 | -0.01    | 0.953** | 19 | 0.148 |
|  |                 |   |       |         |         |    |       |       |          |         |    |       |
|  | fplonci6, (25)  | 1 | 0.988 | -0.007  | 0.91**  | 20 | 0.297 | 0.987 | -0.025** | 0.994** | 13 | n.a.  |
|  |                 | 2 | 0.961 | 0.023   | 0.837** | 22 | 0.584 | 0.952 | 0.025    | 0.812** | 15 | 0.695 |
|  |                 | 3 | 0.712 | 0.001   | 0.75**  | 21 | 1.000 | 0.647 | 0.026    | 0.658** | 19 | 1.559 |
|  |                 | 4 | 0.956 | 0.015   | 0.775** | 23 | 0.871 | 0.94  | -0.033   | 0.950** | 17 | 0.158 |
|  |                 |   |       |         |         |    |       |       |          |         |    |       |
|  | fplonci7, (25)  | 1 | 0.998 | -0.008  | 0.987** | 19 | 0.040 | 0.995 | -0.009   | 0.993** | 14 | 0.021 |
|  |                 | 2 | 0.98  | -0.002  | 0.864** | 25 | 0.472 | 0.987 | 0.009    | 0.816** | 19 | 0.676 |
|  |                 | 3 | 0.934 | 0.004   | 0.905** | 18 | 0.315 | 0.907 | 0.017    | 0.859** | 16 | 0.492 |
|  |                 | 4 | 0.998 | -0.003  | 0.975** | 20 | 0.077 | 0.996 | -0.005   | 0.983** | 18 | 0.052 |
|  |                 |   |       |         |         |    |       |       |          |         |    |       |
|  | fplonci8, (25)  | 1 | 0.994 | 0.005   | 0.94**  | 19 | 0.191 | 0.998 | 0        | 0.964** | 14 | 0.112 |
|  |                 | 2 | 0.919 | -0.046  | 0.858** | 24 | 0.497 | 0.892 | -0.043   | 0.845** | 21 | 0.550 |
|  |                 | 3 | 0.99  | 0       | 0.908** | 23 | 0.304 | 0.988 | -0.002   | 0.908** | 17 | 0.304 |
|  |                 | 4 | 0.733 | 0.016   | 0.602** | 24 | 1.983 | 0.524 | 0.051    | 0.471** | 21 | 3.369 |
|  |                 |   |       |         |         |    |       |       |          |         |    |       |
|  | fplonci9, (25)  | 1 | 0.978 | -0.029  | 0.898** | 17 | 0.341 | 0.97  | -0.057** | 1.017** | 10 | n.a.  |
|  |                 | 2 | 0.996 | 0.006   | 0.853** | 23 | 0.517 | 0.995 | 0.009    | 0.844** | 19 | 0.555 |
|  |                 | 3 | 0.978 | -0.015  | 0.903** | 21 | 0.322 | 0.961 | -0.002   | 0.851** | 15 | 0.525 |
|  |                 | 4 | 0.999 | -0.002  | 0.993** | 17 | 0.021 | 0.999 | -0.003   | 0.997** | 16 | 0.009 |
|  |                 |   |       |         |         |    |       |       |          |         |    |       |
|  | fplonci10, (25) | 1 | 0.974 | 0.046** | 0.748** | 19 | n.a.  | 0.975 | 0.026    | 0.802** | 11 | 0.741 |
|  |                 | 2 | 0.988 | -0.015  | 0.993** | 17 | 0.021 | 0.977 | -0.014   | 0.988** | 12 | 0.036 |
|  |                 | 3 | 0.989 | 0       | 0.954** | 22 | 0.145 | 0.823 | 0.001    | 0.948** | 16 | 0.165 |

|       |                     |   |       |         |         |    |       |       |         |         |    |       |
|-------|---------------------|---|-------|---------|---------|----|-------|-------|---------|---------|----|-------|
|       |                     | 4 | 0.892 | 0.031   | 0.754** | 22 | 0.979 | 0.85  | 0.033   | 0.739** | 19 | 1.060 |
|       |                     |   |       |         |         |    |       |       |         |         |    |       |
| 10(4) | fpbasei(1),<br>(25) | 1 | 0.992 | -0.012  | 0.965** | 19 | 0.109 | 0.988 | -0.018  | 0.988** | 15 | 0.036 |
|       |                     | 2 | 0.835 | -0.027  | 0.869** | 20 | 0.452 | 0.755 | -0.005  | 0.787** | 18 | 0.812 |
|       |                     | 3 | 0.946 | 0.028   | 0.902** | 18 | 0.326 | 0.916 | 0.05    | 0.841** | 12 | 0.567 |
|       |                     | 4 | 0.969 | 0.055** | 0.802** | 17 | n.a.  | 0.958 | 0.033   | 0.866** | 13 | 0.464 |
|       |                     |   |       |         |         |    |       |       |         |         |    |       |
|       | fpbasei(2),<br>(25) | 1 | 0.912 | 0.026   | 0.761** | 20 | 0.942 | 0.881 | 0.035   | 0.704** | 16 | 1.261 |
|       |                     | 2 | 0.979 | -0.003  | 0.877** | 23 | 0.421 | 0.977 | -0.025  | 0.954** | 18 | 0.145 |
|       |                     | 3 | 0.822 | -0.041  | 0.839** | 21 | 0.576 | 0.757 | -0.038  | 0.815** | 17 | 0.681 |
|       |                     | 4 | 0.857 | 0.105** | 0.608** | 18 | n.a.  | 0.774 | 0.130** | 0.504** | 11 | n.a.  |
|       |                     |   |       |         |         |    |       |       |         |         |    |       |
|       | fpbasei(3),<br>(25) | 1 | 0.95  | -0.004  | 0.805** | 23 | 0.727 | 0.901 | 0.004   | 0.767** | 17 | 0.911 |
|       |                     | 2 | 0.997 | -0.007  | 0.982** | 19 | 0.055 | 0.995 | -0.008  | 0.988** | 17 | 0.036 |
|       |                     | 3 | 0.994 | 0.01    | 0.942** | 20 | 0.185 | 0.999 | -0.014  | 0.986** | 12 | 0.043 |
|       |                     | 4 | 0.954 | -0.048  | 1.032** | 15 | 0.001 | 0.9   | -0.047  | 1.028** | 13 | 0.001 |

|      |              |   |       |              |         |    |       |       |              |         |    |       |
|------|--------------|---|-------|--------------|---------|----|-------|-------|--------------|---------|----|-------|
|      |              |   |       |              |         |    |       |       |              |         |    |       |
| 5(5) | fd9', (20)   | 1 | 0.983 | -<br>0.049** | 0.961** | 18 | n.a.  | 0.979 | -<br>0.041** | 0.936** | 15 | n.a.  |
|      |              | 2 | 0.99  | -<br>0.038** | 1.006** | 15 | n.a.  | 0.984 | -0.034       | 0.993** | 13 | 0.028 |
|      |              | 3 | 0.995 | -0.017       | 0.918** | 16 | 0.357 | 0.989 | -0.014       | 0.905** | 11 | 0.420 |
|      |              | 4 | 0.986 | 0.001        | 0.90**  | 15 | 0.444 | 0.984 | -0.008       | 0.925** | 14 | 0.324 |
|      |              | 5 | 0.97  | 0.02         | 0.866** | 19 | 0.619 | 0.959 | 0.03         | 0.832** | 12 | 0.808 |
|      | dfd9, (10)   | 1 | 0.996 | -0.002       | 0.918** | 7  | 0.357 | 0.991 | 0.013        | 0.876** | 5  | 0.566 |
|      |              | 2 | 0.952 | -0.019       | 0.916** | 7  | 0.367 | 0.927 | -0.073       | 1.054** | 6  | 0.001 |
|      |              | 3 | 0.969 | -<br>0.107** | 1.035** | 9  | n.a.  | 0.947 | -0.094       | 0.984** | 6  | 0.065 |
|      |              | 4 | 0.99  | 0.005        | 0.894** | 10 | 0.474 | 0.99  | -0.008       | 0.924** | 8  | 0.329 |
|      |              | 5 | 0.949 | -0.149       | 0.942** | 4  | 0.246 | 0.992 | -0.227       | 1.107** | 3  | 0.001 |
|      |              |   |       |              |         |    |       |       |              |         |    |       |
| 7(6) | dfd2ri, (10) | 1 | 0.994 | -0.015       | 0.984** | 9  | 0.081 | 0.994 | -0.015       | 0.984** | 9  | 0.081 |
|      |              | 2 | 0.998 | -<br>0.023** | 0.957** | 9  | n.a.  | 0.997 | -0.023       | 0.959** | 7  | 0.214 |
|      |              | 3 | 0.956 | 0.037        | 0.753** | 10 | 1.640 | 0.956 | 0.037        | 0.753** | 10 | 1.640 |
|      |              | 4 | 0.939 | -0.007       | 0.882** | 7  | 0.669 | 0.939 | -0.007       | 0.882** | 7  | 0.669 |
|      |              | 5 | 0.962 | 0.004        | 0.826** | 9  | 1.053 | 0.969 | 0.008        | 0.799** | 8  | 1.258 |
|      |              | 6 | 0.827 | -0.047       | 0.699** | 10 | 2.153 | 0.842 | -0.025       | 0.6**   | 9  | 3.333 |
|      | dfd4, (10)   | 1 | 0.995 | -0.027       | 0.968** | 7  | 0.165 | 0.991 | -0.029       | 0.973** | 6  | 0.139 |
|      |              | 2 | 0.65  | -0.043       | 0.805** | 8  | 1.211 | 0.445 | -0.023       | 0.766** | 6  | 1.527 |
|      |              | 3 | 0.974 | -0.04        | 0.926** | 8  | 0.400 | 0.979 | -0.041       | 0.920** | 5  | 0.435 |
|      |              | 4 | 0.996 | -<br>0.024** | 0.942** | 9  | n.a.  | 0.996 | -<br>0.024** | 0.942** | 9  | n.a.  |
|      |              | 5 | 0.936 | 0.032        | 0.773** | 10 | 1.468 | 0.936 | 0.032        | 0.773** | 10 | 1.468 |

|  |             |   |       |        |         |    |       |       |        |         |    |       |
|--|-------------|---|-------|--------|---------|----|-------|-------|--------|---------|----|-------|
|  |             | 6 | 0.997 | -0.017 | 0.964** | 9  | 0.187 | 0.997 | -0.015 | 0.957** | 8  | 0.225 |
|  |             |   |       |        |         |    |       |       |        |         |    |       |
|  | fdf2', (20) | 1 | 0.992 | 0.003  | 0.96**  | 14 | 0.208 | 0.991 | 0.003  | 0.96**  | 12 | 0.208 |
|  |             | 2 | 0.982 | 0.007  | 0.886** | 19 | 0.643 | 0.99  | -0.09  | 0.924** | 15 | 0.411 |
|  |             | 3 | 0.997 | 0.01   | 0.935** | 18 | 0.348 | 0.996 | 0.007  | 0.943** | 17 | 0.302 |
|  |             | 4 | 0.95  | -0.008 | 0.874** | 17 | 0.721 | 0.927 | -0.015 | 0.884** | 12 | 0.656 |
|  |             | 5 | 0.998 | -0.005 | 0.968** | 19 | 0.165 | 0.998 | -0.005 | 0.968** | 19 | 0.165 |
|  |             | 6 | 0.997 | -      | 0.966** | 19 | n.a.  | 0.997 | -      | 0.966** | 19 | n.a.  |
|  |             |   |       |        |         |    |       |       |        |         |    |       |
|  | fdf4', (20) | 1 | 0.902 | 0.031  | 0.751** | 17 | 1.658 | 0.902 | 0.031  | 0.751** | 17 | 1.658 |
|  |             | 2 | 0.989 | -0.021 | 0.931** | 17 | 0.371 | 0.989 | -0.021 | 0.931** | 17 | 0.371 |
|  |             | 3 | 0.974 | -0.032 | 0.928** | 17 | 0.388 | 0.956 | -0.02  | 0.896** | 13 | 0.580 |
|  |             | 4 | 0.942 | -0.002 | 0.78**  | 19 | 1.410 | 0.942 | -0.002 | 0.780** | 19 | 1.410 |
|  |             | 5 | 0.964 | -0.01  | 0.873** | 16 | 0.727 | 0.964 | -0.01  | 0.873** | 16 | 0.727 |
|  |             | 6 | 0.993 | -0.016 | 0.976** | 18 | 0.123 | 0.988 | -0.015 | 0.969** | 14 | 0.160 |

|      |             |   |       |          |         |    |       |       |          |         |    |       |
|------|-------------|---|-------|----------|---------|----|-------|-------|----------|---------|----|-------|
| 8(9) | fdf5', (20) | 1 | 0.995 | -0.017   | 1.000** | 17 | 0.001 | 0.995 | -0.017   | 1.000** | 17 | 0.001 |
|      |             | 2 | 0.969 | -0.085** | 1.025** | 18 | n.a.  | 0.943 | -0.081** | 1.01**  | 14 | n.a.  |
|      |             | 3 | 0.992 | -0.052** | 0.984** | 18 | n.a.  | 0.99  | -0.049** | 0.973** | 16 | n.a.  |
|      |             | 4 | 0.995 | -0.01    | 0.959** | 18 | 0.342 | 0.996 | -0.025** | 0.997** | 14 | n.a.  |
|      |             | 5 | 0.998 | -0.001   | 0.971** | 17 | 0.239 | 0.999 | -0.007** | 0.989** | 16 | n.a.  |
|      |             | 6 | 0.994 | -0.026** | 0.984** | 20 | n.a.  | 0.995 | -0.026** | 0.981** | 17 | n.a.  |
|      |             | 7 | 0.999 | -0.004   | 0.990** | 14 | 0.081 | 0.999 | -0.003   | 0.988** | 13 | 0.097 |
|      |             | 8 | 0.992 | 0.003    | 0.946** | 17 | 0.457 | 0.992 | 0.003    | 0.947** | 16 | 0.448 |
|      |             | 9 | 0.999 | -0.015** | 1.013** | 20 | n.a.  | 0.999 | -0.015** | 1.013** | 20 | n.a.  |
|      |             |   |       |          |         |    |       |       |          |         |    |       |
|      | dfd5, (10)  | 1 | 0.994 | 0.001    | 0.969** | 10 | 0.256 | 0.996 | -0.011   | 0.998** | 9  | 0.016 |
|      |             | 2 | 0.986 | 0.026    | 0.917** | 10 | 0.724 | 0.999 | -0.006   | 0.978** | 8  | 0.180 |
|      |             | 3 | 0.999 | -0.022** | 0.948** | 8  | n.a.  | 0.999 | -0.022** | 0.948** | 8  | n.a.  |
|      |             | 4 | 0.953 | -0.141** | 1.126** | 10 | n.a.  | 0.953 | -0.141** | 1.126** | 10 | n.a.  |
|      |             | 5 | 0.993 | -0.014   | 0.962** | 10 | 0.316 | 0.99  | -0.009   | 0.952** | 9  | 0.403 |
|      |             | 6 | 0.989 | -0.024   | 0.951** | 10 | 0.412 | 0.989 | -0.024   | 0.951** | 10 | 0.412 |
|      |             | 7 | 0.995 | -0.04**  | 0.990** | 10 | n.a.  | 0.992 | -0.041** | 0.991** | 9  | n.a.  |
|      |             | 8 | 0.994 | -0.861   | 0.971** | 6  | 0.239 | 0.998 | 0.014    | 0.951** | 6  | 0.412 |
|      |             | 9 | 0.959 | -0.136** | 1.19**  | 10 | n.a.  | 0.959 | -0.136** | 1.19**  | 10 | n.a.  |

Note: For the subjects whose  $\hat{\tau}_i < 0$  which shows an extremely risk averse attitude, we truncate the corresponding estimated risk parameters to 0.